# ABSTRACTS OF PAPERS

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**1. Random Variables with Comparable Peakedness.** Z. W. Birnbaum, University of Washington.

Let $U$ and $V$ be random variables with symmetrical distributions, i.e. with $P(U \leqq -T) = P(U \geqq T)$ and $P(V \leqq -T) = P(V \geqq T)$ for all $T \geqq 0$. The random variable $U$ shall be called more peaked than $V$ if $P(| U | \geqq T) \leqq P(| V | \geqq T)$ for all $T \geqq 0$. Let $X_1, Y_1$ and $X_2, Y_2$ be two pairs of independent random variables such that $X_i$ is more peaked than $Y_i$ for $i = 1, 2$. Then under certain additional conditions $X = X_1 + X_2$ is more peaked than $Y = Y_1 + Y_2$.

**2. On Optimum Tests of Composite Hypotheses with One Constraint.** Erich L. Lehmann, University of California, Berkeley.

The problem studied is that of finding all similar and bisimilar test regions of composite hypotheses, and of obtaining the most powerful of these regions. Various results are obtained for distributions which admit sufficient statistics with respect to their parameters. Applications are made to the hypothesis specifying the value of the circular correlation coefficient in a normal population, and certain hypotheses concerning scale and location parameters in exponential and rectangular populations.

**3. Estimation of a Distribution Function by Confidence Limits.** Frank J. Massey, Jr., University of California, Berkeley.

Let $x_1, x_2, \cdots, x_n$ be the results of $n$ independent observations, having the same cumulative distribution function $F(x)$. Form the function $S_n(x) = k/n$ where $k$ is the number of observations less than or equal to $x$. A confidence band $S_n(x) \pm \lambda/\sqrt{n}$ will be used to estimate $F(x)$. To determine the confidence coefficient it is necessary to find $Pr\{\max \sqrt{n} \mid S_n(x) - F(x) \mid \leqq \lambda/\sqrt{n}\}$. It is sufficient to consider $x$ uniformly distributed in the interval $(0, 1)$. Let $\lambda\sqrt{n} = s/t$ where $s$ and $t$ are integers. Then $S_n(x)$, to stay in the band $F(x) \pm \lambda/\sqrt{n}$, can only pass through certain lattice points above $x = i/tn$, $i = 1, 2, \cdots, tn$. The probability of $S_n(x)$ passing through a particular sequence of these points is given by the multinomial law, and this can be summed over all permissible sequences. Limiting distributions have been given by A. Kolmogoroff, and by N. Smirnoff. It is desired to test the hypothesis $F(x) = F_0(x)$ against alternatives $F(x) = F_1(x)$. Using the criterion: reject $F_0(x)$ if

$$\max_x \sqrt{n} \mid F_0(x) - S_n(x) \mid > \lambda$$

the probability of first kind of error can be controlled by choice of $\lambda$. A lower bound to the probability of second kind of error against alternatives such that $\max \sqrt{n} \mid F_0(x) - F_1(x) \mid \geqq \Delta$ is given. This lower bound approaches one as $n \to \infty$. Thus the test is consistent.

**4. A Note on Sequential Confidence Sets.** Charles Stein, Columbia University.

This paper generalizes a paper of Stein and Wald, appearing in the *Annals of Math. Stat.*, Sept., 1947.

Let $\{X_i\}$, $(i = 1, 2, \cdots)$, be a sequence of random variables whose distribution depends on an unknown parameter $\theta$. Sequential confidence sets are determined by a rule indicating