Substituting (34) in (32), and equating coefficients of like powers of (x, y), we obtain the recursion formulae

$$(35)\quad \sum_{j+k=n} B_{ij}B_{0k}[j][2k-j+1] = \sum_{j+k=n-1} B_{i+1,j}B_{0k}[i+1][j-k]; \quad i: 0, 1, \cdots.$$

From (10), it is readily verified that $B_{i0} = 0$ for $i \neq 0$, so that equations (35) give solutions for the B_{ij} in terms of the B_{0k}. These solutions are of interest since they show a one-to-one correspondence between the functions $G(0, y)$ and $G(x, y)$, for $(x, y) \in [R \cap S]$.

NUMERICAL INTEGRATION FOR LINEAR SUMS OF EXPONENTIAL FUNCTIONS

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1. Introduction. The methods of numerical integration going by the names trapezoidal rule, Simpson's rule, Weddle's rule, and the Newton-Cotes formulae are of the type

$$(1)\qquad \int_{-1}^{1} f(x)\,dx \simeq \sum_{i=0}^{n} \lambda_{in} f(x_{in})$$

where the abscissae $\{x_{in}\}$ are uniformly distributed on a finite interval, chosen as $(-1, 1)$ for convenience,

$$(2)\qquad x_{in} = -1 + \frac{2i}{n}, \qquad i = 0, 1, 2, \cdots, n,$$

and where the set of constants $\{\lambda_{in}\}$ depend on the name of the rule and the value of n but not on the function $f(x)$. Throughout this note all abscissae will be assumed to be uniformly distributed on $(-1, 1)$ unless the contrary is explicitly stated.

Since correspondence relation (1) involves $(n + 1)$ constants $\{\lambda_{in}\}$, it might be possible to choose $(n + 1)$ arbitrary functions $g_j(x)$, $j = 0, 1, 2, \cdots, n$, and require that the set $\{\lambda_{in}\}$ be the solution, if such exists, of the $(n + 1)$ simultaneous linear equations

$$(3)\qquad \int_{-1}^{1} g_j(x)\,dx = \sum_{i=0}^{n} \lambda_{in} g_j(x_{in}), \qquad j = 0, 1, 2, \cdots, n.$$

Indeed, the selection

$$(4)\qquad g_j(x) = x^j, \qquad j = 0, 1, 2, \cdots, n,$$

will give a set of $(n + 1)$ simultaneous equations of form (3) and the solution $\{\lambda_{in}\}$ is the set of Newton-Cotes weights for that value of n. The numerical evaluation

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