A SEQUENTIAL DECISION PROCEDURE FOR CHOOSING ONE OF THREE HYPOTHESES CONCERNING THE UNKNOWN MEAN OF A NORMAL DISTRIBUTION

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1. Introduction. In this paper a multi-decision problem is investigated from a sequential viewpoint and compared with the best non-sequential procedure available. Multi-decision problems occur often in practice but methods to deal with such problems are not yet sufficiently developed.

The problem under consideration here is a 3-decision problem: Given a chance variable which is normally distributed with known variance σ^2, but unknown mean θ, and given two real numbers $a_1 < a_2$, the problem is to choose one of the three mutually exclusive and exhaustive hypotheses

$$H_1 : \theta < a_1 \qquad H_2 : a_1 \leqq \theta \leqq a_2 \qquad H_3 : \theta > a_2 .$$

In order to select a proper sequential decision procedure, the parameter space is subdivided into 5 mutually exclusive and exhaustive zones in the following manner. Around a_1 there exists an interval (θ_1, θ_2) in which we have no strong preference between H_1 and H_2 but prefer (strongly) to reject H_3. Around a_2 there exists an interval (θ_3, θ_4) in which we have no strong preference between H_2 or H_3 but prefer (strongly) to reject H_1. For $\theta \leqq \theta_1$ we prefer to accept H_1. For $\theta_2 \leqq \theta \leqq \theta_3$ we prefer to accept H_2. For $\theta \geqq \theta_4$ we prefer to accept H_3.

The intervals (θ_1, θ_2) and (θ_3, θ_4) will be called indifference zones. The determination of these indifference zones is not a statistical problem but should be made on practical considerations concerning the consequences of a wrong decision.

In accordance with the above we define a wrong decision in the following way. For $\theta \leqq \theta_1$, acceptance of H_2 or H_3 is wrong. For $\theta_1 < \theta < \theta_2$ acceptance of H_3 is wrong. For $\theta_2 \leqq \theta \leqq \theta_3$, acceptance of H_1 or H_3 is wrong. For $\theta_3 < \theta < \theta_4$, acceptance of H_1 is wrong. For $\theta \geqq \theta_4$, acceptance of H_1 or H_2 is wrong.

The requirements on our decision procedure necessary to limit the probability of a wrong decision are investigated. Two cases are considered.

Case 1: Prob. of a wrong decision $\leqq \gamma$ for all θ.

$$\text{Case 2:} \begin{cases} \text{Prob. of a wrong decision} \leqq \gamma_1 \text{ for } \theta \leqq \theta_1, \\ \text{Prob. of a wrong decision} \leqq \gamma_2 \text{ for } \theta_1 < \theta < \theta_4, \\ \text{Prob. of a wrong decision} \leqq \gamma_3 \text{ for } \theta \geqq \theta_4. \end{cases}$$

The decision procedure discussed in the present paper is not an optimum procedure since, as will be seen later, the final decision at the termination of

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