For $n = 2$ and $a_1 = a_2 = b_1 = 1$, $b_2 = -1$ we obtain from (22)

$$f_s(t) = \exp\left[-\left(\sigma_1^2 + \sigma_2^2\right)t^2/2\right] \qquad s = 1, 2.$$

This shows that $\sigma_1^2 = \sigma_2^2$ and establishes Bernstein's theorem.

## REFERENCES

[1] S. Bernstein, "Sur une proprieté caractéristique de la loi de Gauss, *Trans. Leningrad Polytechnic Institute* (1941).

[2] H. Cramér, "Über Eine Eigenschaft Der Normalen Verteilungsfunktion," *Math. Z.* Vol. 41 (1936), pp. 405–414.

[3] M. Fréchet, "Généralisation de la loi de probabilité de Laplace, *Ann. Inst. H. Poincaré*, Vol. 13 (1951), pp. 1–29.

[4] M. Kac, "A characterization of the normal distribution," *Amer. J. Math.*, Vol. 61 (1939), pp. 726–728.

[5] Yu. V. Linnik, "Remarks concerning the classical derivation of Maxwell's law," *Doklady Akad. Nauk SSSR*, Vol. 85 (1952), pp. 1251–1254.

[6] J. Marcinkiewicz, "Sur une proprieté de la loi de Gauss," *Math. Z.* Vol. 44 (1938), pp. 612–618.

## ADDENDUM

The authors are indebted to Professor G. Darmois for calling their attention to his note in the *C. R. Acad. Sci. Paris*, Vol. 232 (1951), pp. 1999–2000 in which he proved the theorem for $n = 2$ without assuming the existence of moments. He later extended this to the case of arbitrary $n$. His paper will be published in the *Bulletin of the International Statistical Institute*. The method of proof used by Professor Darmois is different from the one presented in this paper. The authors learned that these results were also obtained by methods similar to Darmois' by B. V. Gnedenko (*Izvestiya Akad. Nauk. SSSR, Ser. Mat.*, Vol. 12 (1948), pp. 97–100) for the case $n = 2$ and by V. P. Skitovich (*Doklady Akad. Nauk. SSSR* (N.S.) Vol. 89 (1953), pp. 217–219) for any $n$.

While reading the proofs of this paper the authors learned that the theorem was also discussed by M. Loève in the appendix to P. Lévy's "Processus stochastiques", Gauthier-Villars, Paris, 1948, pp. 337–338.

# ON OPTIMAL SYSTEMS[1]

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**1. Summary.** For any sequence $x_1, x_2, \cdots$ of chance variables satisfying $|x_n| \leqq 1$ and $E(x_n \mid x_1, \cdots, x_{n-1}) \leqq -u(\max |x_n| \mid x_1, \cdots, x_{n-1})$, where $u$ is a fixed constant, $0 < u < 1$, and for any positive number $t$,

$$\Pr\left\{\sup_n (x_1 + \cdots + x_n) \geqq t\right\} \leqq \left(\frac{1-u}{1+u}\right)^t.$$

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