

# ESTIMATION BY THE MINIMUM DISTANCE METHOD IN NONPARAMETRIC STOCHASTIC DIFFERENCE EQUATIONS<sup>1</sup>

By J. WOLFOWITZ

*Cornell University*

**1. Introduction.** The present paper is intended to report some of the ideas described in a special invited address delivered by the author at the meeting of the Institute of Mathematical Statistics at Chicago on December 29, 1952. This address dealt with two topics: a) the connection between the method of maximum likelihood and the Wald theory of decision functions, with an explanation of the asymptotic efficiency of the former; and b) estimation by the minimum distance method. The first of these topics is discussed in [1], and this paper will be devoted to a discussion of the second.

The origin of the minimum distance method is to be found in [2]. Applications of the method were extended and generalized in [3]. The paper [4] contains a theorem which is an essential tool. A paper by Kac, Kiefer, and the present author, entitled "On tests of normality and other tests of goodness of fit based on the minimum distance method," is in preparation.

The method of estimation to which this paper is devoted is characterized by the fact that the estimators are always such as to minimize the distance between suitably chosen distribution functions (d.f.). In a variety of problems, which includes many where classical methods, like that of maximum likelihood, fail to give consistent estimators, it yields estimators which actually converge with probability one to the quantities being estimated; we call such estimators super-consistent. The problems treated in the present paper provide examples of this.

The basic ideas of the proofs of the super-consistency of these estimators are to be found in [2] and [4]. Application of the minimum distance method, unlike that of the method of maximum likelihood, is not mechanical, and, in the cases we have treated, always requires the development of special results.

The present paper presents results on problems not hitherto treated in the literature. It is intended to be largely self-contained, and its organization is as follows. Section 2 gives essential preliminaries. Section 3 contains a statement of some of the results already obtained elsewhere. In Section 4 are formulated three new problems in nonparametric stochastic difference equations. In Sections 5, 6, and 7 we exhibit minimum distance estimators for these problems. In Sections 5 and 6 we prove the super-consistency of the first two estimators.

In a few places the proofs are not given in all detail in the interest of brevity,

---

Received 6/24/53.

<sup>1</sup> This research was supported in part by the United States Air Force under Contract No. AF18(600)-685 monitored by the Office of Scientific Research.

*Editor's Note:* This paper was presented to the Chicago meeting of the Institute of Mathematical Statistics, December 29, 1952, and is published in the *Annals* by invitation of the Institute Committee on Special Invited Papers.