PROOF. Say X has density p with respect to Lebesgue measure on the unit interval. Then

$$U_k(\lambda) = \lambda\, d(y_k(X), \lambda D^{-k})/d(y_k(X), D^{-k}),$$

where $y_k(s) = mD^{-k}$ for $mD^{-k} \leqq s < (m+1)D^{-k}$, $m = 0, 1, \cdots, D^k - 1$, and $d(a, h) = h^{-1} \int_a^{a+h} p(s)\, ds$.

We must show that

$$\lambda d(y_k(s), \lambda D^{-k})/d(y_k(s), D^{-k}) \to \lambda$$

for almost all s (Lebesgue measure) for which $p(s) > 0$, and this will follow from

$$d(y_k(s), \lambda D^{-k}) \to p(s) \quad \text{a.e.} \tag{1}$$

Now a basic theorem of real variable theory asserts that

$$d(s, h) \to p(s) \quad \text{a.e.} \tag{2}$$

as $h \to 0$. Let $a_k(s) = (s - y_k(s))/\lambda D^{-k}$

Then

$$\begin{aligned} d(y_k(s), \lambda D^{-k}) &= a_k(s)\, d(s, y_k(s) - s) + [1 - a_k(s)]\, d(s, y_k(s) + \lambda D^{-k} - s) \\ &= a_k(s)[d(s, y_k(s) - s) - d(s, y_k(s) + \lambda D^{-k} - s)] \\ &\qquad + d(s, y_k(s) + \lambda D^{-k} - s). \end{aligned} \tag{3}$$

Since $a_k(s)$ is bounded, letting $k \to \infty$ in (3) and using (2) yields (1), and the proof is complete.

REFERENCE

[1] T. E. HARRIS, "On chains of infinite order," *Pacific J. Math.*, Vol. 5 (1955), pp. 707–724.

A PROOF THAT THE SEQUENTIAL PROBABILITY RATIO TEST (S.P.R.T.) OF THE GENERAL LINEAR HYPOTHESIS TERMINATES WITH PROBABILITY UNITY

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1. Introduction. It can be shown [1] [2] that the S.P.R.T. of the general linear hypothesis resolves itself into the following form of procedure: Continue sampling at stage (n) if

$$\frac{\beta}{1-\alpha} < e^{-\lambda^{(n)}/2} M\left(\alpha(n), \gamma; \frac{\frac{1}{2}\lambda^{(n)} G^{(n)}}{1 + G^{(n)}}\right) < \frac{1-\beta}{\alpha} \cdots; \tag{1}$$

otherwise accept or reject the null hypothesis depending upon whether the left-hand or right-hand inequality is violated.

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