

with the two branches of this hyperbola. It is readily seen that the condition that such an  $x$ -interval exists and be of finite length is equivalent to the condition that the two asymptotes of the hyperbola have slopes of equal sign. Since these slopes are  $b_1 - K/[\sum(x - \bar{x})^2]^{\frac{1}{2}}$  and  $b_1 + K/[\sum(x - \bar{x})^2]^{\frac{1}{2}}$ , the condition in question is  $b_1^2 - K^2/(\sum(x - \bar{x})^2) > 0$ . This is condition (10) obtained previously by a different line of reasoning.

It may be observed, finally, that the inverse problem, viz, to determine uncertainty intervals for observed  $y$  values corresponding to given  $x$  values [2] is not a classical case of interval estimation, since it is concerned with bracketing a random variable, not a population parameter, by means of two statistics. Intervals of this type are discussed by Weiss [7].

Applications of the procedure outlined in this note to a problem in chemistry are discussed elsewhere [5].

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### A NOTE ON INCOMPLETE BLOCK DESIGNS

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**1. Introduction.** Kempthorne [1] has shown the efficiency factor of an incomplete block design to be a quantity proportional to the harmonic mean of the non-zero latent roots of the matrix of coefficients of the reduced normal equations for the intra-block estimates of treatment effects. He has further stated that the geometric mean in a certain sense corresponds to the generalized variance but has not explicitly explained it. The present note is intended to clear this point and to prove that the design with highest efficiency factor (in any case, whether the harmonic mean or the geometric mean is taken as a measure of efficiency) is (a) a balanced incomplete block design, if such a design exists; and

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