SOME CONTRIBUTIONS TO ANOVA IN ONE OR MORE DIMENSIONS: II

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0. Introduction and notation. This paper presents certain natural extensions, to the multi-dimensional or multivariate situation, of the results contained in the first paper [10] by the authors. We shall use the same notation as before and, in addition, we shall use the following notation: $c(A)$ will denote all the characteristic roots of the matrix A, and if A is at least positive semi-definite, then $c_{\min}(A)$ and $c_{\max}(A)$ will denote, respectively, the smallest and the largest of these roots; $D_a(p \times p)$ will denote a diagonal matrix whose elements are a_1, a_2, $\cdots$, a_p; $\tilde{T}(p \times p)$ will denote a triangular matrix whose non-zero elements are along and below the diagonal; $|A|$ will denote the determinant of a square matrix A; and, $A(p \times p)\cdot\times B(q \times q)$ will denote the Kronecker product or right direct product [5] of the matrices A and B. Also min (p, q) will denote the lesser of the two real numbers p and q.

1. Resume of problems and results under the multivariate Model I of ANOVA.

1.1 *The multivariate Model I*: Let $X(p \times n) = p[\mathbf{x}_1\, \mathbf{x}_2 \cdots \mathbf{x}_n]$ be a set of n observable stochastic p-vectors such that

$$X'(n \times p) = A(n \times m)\xi(m \times p) + \epsilon(n \times p), \qquad m < n,$$

$$= \begin{array}{cc} n & \left[\begin{array}{cc} A_I & A_D \end{array}\right] \\ & \begin{array}{cc} r & (m-r) \end{array} \end{array} \begin{array}{c} \left[\begin{array}{c} \xi_I \\ \xi_D \end{array}\right] \\ p \end{array} \begin{array}{c} r \\ (m-r) \end{array} + \epsilon(n \times p) \quad \text{(say)}, \tag{1.1.1}$$

where, as in the univariate situation, A is the *design* matrix with rank $(A) = r \leqq m \leqq n$, and A_I is a basis of A with a consequent partitioning of ξ into $\xi_I(r \times p)$ and $\xi_D(\overline{m - r} \times p)$, and where

(i) $\xi(m \times p)$ is a set of unknown parameters;

(ii) $\epsilon(n \times p)$, whose elements are physically of the nature of errors, is a random sample of size n from the non-singular p-variate normal $N[\mathbf{O}(p \times 1), \Sigma(p \times p)]$. Furthermore, we assume here that $p \leqq (n - r)$.

Under this model it is seen that $\mathbf{x}_i(p \times 1)$, for $i = 1, 2, \cdots, n$, are n independent stochasic p-vectors such that $\mathbf{x}_i$ is $N[E(\mathbf{x}_i), \Sigma]$, where the unknown dispersion matrix $\Sigma(p \times p)$ is the same for all the n vectors, and $E(\mathbf{x}_i)$, for $i = 1, 2, \cdots, n$, is given by

$$E[X'](n \times p) = A(n \times m)\xi(m \times p) = [A_I\; A_D]\begin{bmatrix} \xi_I \\ \xi_D \end{bmatrix}. \tag{1.1.2}$$

Received July 24, 1957.

[1] This research was sponsored by the United States Air Force through the Office of Scientific Research of the Air Research and Development Command.