THE ADMISSIBILITY OF PITMAN'S ESTIMATOR OF A SINGLE LOCATION PARAMETER[1]

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1. Introduction. Pitman [1] gave a thorough discussion of the problem of estimating the location and scale parameters of a distribution which is known except for one or both of these parameters. In particular, if $X_1 \cdots X_n$ are real random variables independently and identically distributed according to the density $r(x - \xi)$ (with respect to Lebesgue measure), where ξ is unknown but the function r is known, Pitman shows that the estimator

$$\hat{\xi}_0(X_1, \cdots, X_n) = \frac{\int \xi \prod r(X_i - \xi)\, d\xi}{\int \prod r(X_i - \xi)\, d\xi} \tag{1.1}$$

is the best translation-invariant estimator in the sense that it minimizes $\mathcal{E}_\xi[\hat{\xi}(X_1 \cdots X_n) - \xi]^2$ among all estimators $\hat{\xi}$ for which

$$\hat{\xi}(x_1 + c, \cdots, x_n + c) = \hat{\xi}(x_1, \cdots, x_n) + c \tag{1.2}$$

for all $x_1, \cdots, x_n$ and c. Girshick and Savage [2] showed that $\hat{\xi}_0$ is minimax in the class of all estimators (not restricted by (1.2)) and this also follows from the later more general results of Kudo [3] and Kiefer [4]. Karlin [5] has shown that under certain conditions $\hat{\xi}_0$ is admissible, that is, if $\hat{\xi}$ is any estimator for which

$$E_\xi(\hat{\xi}(X_1, \cdots, X_n) - \xi)^2 \leqq E_\xi(\hat{\xi}_0(X_1, \cdots, X_n) - \xi)^2 \tag{1.3}$$

for all ξ, then equality holds for all ξ. Since his conditions are fairly strong, and his method somewhat special, it seems desirable to present an alternative proof. Theorem 1 of Section 2, when reformulated for the present slightly special case, becomes

Theorem. *If*

$$\int \prod r(x_i) \left\{ \frac{\int \xi^2 \prod r(x_i - \xi)\, d\xi}{\int \prod r(x_i - \xi)\, d\xi} - \left(\frac{\int \xi \prod r(x_i - \xi)\, d\xi}{\int \prod r(x_i - \xi)\, d\xi} \right)^2 \right\}^{3/2} \prod dx_i < \infty \tag{1.4}$$

then $\hat{\xi}_0$ *defined by* (1.1) *is admissible.*

Received August 25, 1958.

[1] This work was sponsored by the Office of Naval Research.

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