MOMENTS OF THE ABSOLUTE DIFFERENCE AND ABSOLUTE DEVIATION OF DISCRETE DISTRIBUTIONS[1]

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1. Introduction. Johnson [3], Crow [1] and Ramasubban [5] have discussed the evaluation of the mean difference and the mean deviation for some positive integral valued discrete distributions. These are particular cases of a more general statistic which may be defined as

$$\Delta_r = E \mid X_1 - X_2 \mid^r, \tag{1a}$$

where X_1 and X_2 are two random variables with given distributions. Statistic (1a) will be referred to as the rth moment of the absolute difference of X_1 and X_2. In this paper, Δ_r is evaluated when X_1 and X_2 are independent and both have distributions—possibly different ones—within one of the following families of distributions: (i) Poisson, (ii) Pascal, and (iii) Binomial. The case when X_1 and X_2 are distributed as two independent Logarithmic variables, and the cases when X_1 and X_2 are independent and have distributions in two different families of distributions (chosen from the Poisson, Pascal, Binomial, and Logarithmic families), can be treated along similar lines, but the results are not given here in order to conserve space. Methods are also given to evaluate Δ_r when X_2 is a fixed constant and when X_1 is distributed as (i) a Poisson (ii) a Pascal (iii) a Binomial (iv) a Hypergeometric and (v) a Logarithmic random variable. In this special case, Δ_r will be called the rth moment of the absolute deviation of X_1 about X_2 and denoted by δ_r. I am investigating two sample tests, based on the sample analogues of the Δ_r's, that may be appropriate when the two samples are from two specified but different parametric populations.

2. An expression for the rth moment of the absolute difference $\mid X_1 - X_2 \mid$. Let X_1 and X_2 be two arbitrary independent positive integral valued random variables with probabilities $P_i^{(1)}$ and $P_i^{(2)}$ of obtaining $X_1 = i$ and $X_2 = i$ respectively. Then the rth moment Δ_r is given by

$$\begin{aligned} \Delta_r &= E \mid X_1 - X_2 \mid^r \\ &= \sum_{ik} k^r P\{X_2 - X_1 = k \mid X_1 = i\} P\{X_1 = i\} \\ &\quad + \sum_{ik} k^r P\{X_1 - X_2 = k \mid X_2 = i\} P\{X_2 = i\} \\ &= \sum_{ik} k^r P_i^{(1)} P_{i+k}^{(2)} + \sum_{ik} k^r P_i^{(2)} P_{i+k}^{(1)} \end{aligned} \tag{1}$$

where the summations are over $1, 2, 3, \cdots$.

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