SAMPLING INSPECTION AS A MINIMUM LOSS PROBLEM

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Introduction. In March 1958, in a lecture at Berkeley, Milton Friedman pointed out that statisticians, when asked to recommend a sampling inspection plan for the producer or buyer of a mass product, usually ask him questions which he cannot answer. They first ask: What percentage of defectives would you allow without rejecting the product? If the answer is p_0, the statistician would choose a smaller percentage p_1 and a larger one p_2, and ask questions like: Would you allow a probability of 5 per cent of rejecting the product, if the true percentage is p_1? The answers usually are mere guesses.

However, a competent manager could answer questions like: What is the cost of inspecting a sample of n? What would be your profit or loss if you buy or sell a lot with defective fraction p? What do you do if you reject the product, and what would be your loss in this case? Would it be very expensive to improve the quality of your product? A reasonable production and inspection plan ought to be based solely on these loss functions.

In what follows, we shall leave aside production problems. We shall assume a plant to produce a product of variable quality, the variations being due to accidents we cannot prevent. The only thing the producer can do is to inspect a sample and, if it contains too many defectives, to examine the whole lot and to eliminate the defectives. And the only thing the buyer can do is to inspect a sample and, if it contains too many defectives, to return the product to the producer.

The loss functions will be assumed to be linear functions of the defective fraction p, and the inspection cost to be proportional to the size n of the sample. We shall assume that the same inspection plan is used every day, or in the buyer's case every time he buys a lot, so that in the long run only the average loss counts.

In Sections 1–3 and in Section 4, the minimum loss problem will be discussed from the producer's and from the buyer's point of view separately. In Section 5, the producer's and the buyer's point of view will be combined. It will be shown that the two partners may increase their joint profit by forming a coalition and combining their inspection plans into one.

After having finished an earlier draft of this paper, I learned that S. Moriguti [1] and S. Ura [2] investigated the problem of minimax inspection plans from exactly the same point of view. Moriguti's results are just the same as mine obtained in Section 2 for Case A (p_0n and q_0n large). Ura's results are close to mine obtained in Section 3 for Case B (n large, p_0n not large), but there are slight differences in the numerical values. On the other hand, Ura treated the

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