SLIPPAGE PROBLEMS[1]

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1. Introduction. Slippage problems have been considered in the literature by Mosteller [6], Paulson [8], Truax [11], Doornbos and Prins [2], Kudo [5], and others. Roughly, the problem is as follows: We wish to compare n populations which have density functions $f(x, \theta_1), f(x, \theta_2), \cdots, f(x, \theta_n)$. On the basis of a sample from each population we want to decide if all the θ_i are equal, or, if not, which is the largest. Actually, a more restricted problem is considered in this paper, in which either all parameter values are equal, or all but one are equal and the exceptional one is larger. If the ith one is larger we will say it has slipped to the right. These slippage problems have certain similarities with the problem of ranking means considered by Bechhofer and others [1], but differ in that the latter deal mostly with procedures guaranteeing with prescribed probability the selection of the population with the largest parameter, where it is known in advance that one parameter exceeds all the others. These authors never allow the possibility that all parameters of the various populations are equal, which, in our situation, is called hypothesis zero. Other contrasts between the two problems will become apparent in our later discussions.

A slightly different problem can be formulated in which we have in addition a control population. The problem is then to compare the n populations with the control, and decide if all the parameters are equal to the parameter of the control population, or, if not, which of the n populations has the larger parameter. In order to obtain optimal solutions to the slippage problems, certain invariance restrictions will be imposed. Notice the obvious symmetry that states, if $X_1, X_2, \cdots, X_n$ is observed (X_i is an observation from the ith population) and if action j is appropriate (i.e., the jth parameter has slipped to the right) then if a permutation $X_{\pi 1}, X_{\pi 2}, \cdots, X_{\pi n}$ is observed, action πj is appropriate. This suggests restricting attention to symmetric procedures. That is, if $\varphi_i(X_1, X_2, \cdots, X_n)$ denotes the probability of taking action i when $X_1, X_2, \cdots, X_n$ is observed, then we will require $\varphi_{\pi i}(X_{\pi 1}, X_{\pi 2}, \cdots, X_{\pi n}) = \varphi_i(X_1, X_2, \cdots, X_n)$ for all permutations $(1, 2, \cdots, n) \rightarrow (\pi 1, \pi 2, \cdots, \pi n)$. We will further restrict attention exclusively to those problems in which it is possible to reduce the problem, by invariance, to a one parameter problem. In particular we will investigate several cases where the parameter is a translation or scale parameter.

The nature of the Bayes solutions will be examined for these problems. The Bayes solutions are usually fairly easy to characterize, and many problems lead us to complete classes of solutions. We will show that any symmetric Bayes solu-

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