LIMITING DISTRIBUTION OF THE MAXIMUM TERM IN SEQUENCES OF DEPENDENT RANDOM VARIABLES[1]

By Simeon M. Berman

Columbia University

Introduction. This paper contains an investigation of the limiting distribution of the maximum term in sequences of random variables subject to certain kinds of dependence.

The limiting distribution of the maximum term in a sequence of independent random variables with a common distribution was completely analyzed in a series of works by many writers; this research culminated in the comprehensive paper of Gnedenko [5]. The assumption of a common distribution was dropped by Juncosa [7]. Watson proved that under mild restrictions, the limiting distribution of the maximum in an m-dependent stationary sequence is the same as that for independent random variables with a common distribution [11]. A complete bibliography is contained in the book by Gumbel [6].

The first section of this paper contains a brief review of the classical case of independent and identically distributed random variables as given in Gnedenko's paper [5].

The present work generalizes the classical theory.

1. In the second section, the maximum term in a sequence of exchangeable random variables is considered. The limiting distribution is a mixture of the distributions obtained in the case of independent random variables. This is analogous to the findings of Blum, Chernoff, Rosenblatt, and Teicher [2] and Buhlmann [3], who discovered that the limiting distributions of the *sums* of exchangeable random variables are mixtures of normal distributions.

2. The second generalization is the case where the number of random variables considered in the determination of the maximum is itself a random variable N_n, depending on a nonnegative, integer-valued parameter n. If the sequence $\{N_n\}$ is distributed independently of the observed random variables, and if $N_n \xrightarrow{\text{pr}} \infty$ as $n \to \infty$, then the limiting distribution of the maximum is a mixture of the kind described in the previous paragraph. This is similar to the result of Robbins that the limiting distribution of the sum of a random number of random variables is a mixture of normal distributions [10].

A theorem is also stated for the case where the sequence $\{N_n\}$ may depend on the observed random sequence; an analogous theorem for sums has also been given by Anscombe [1].

Any result obtained for the maximum term is also valid, with appropriate modifications, for the minimum term.

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