THE POISSON TENDENCY IN TRAFFIC DISTRIBUTION[1]

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1. Introduction. This note is concerned with models for one-way flow of traffic on the infinite line. A simple and frequently used model is: at $t = 0$, points on the line (automobiles) are selected in accord with a Poisson process. Then velocities are assigned to these points, independently for each point and independent of the position of the point, drawn from the same parent distribution. The points then maintain their assigned velocities at all times. If a fixed point on the line is selected, then times between successive arrivals of autos at this point are, according to the above model, independent random variables with a negative exponential distribution. This result is hardly in agreement with experiment. Frank Haight, of the U.C.L.A. Traffic Engineering Department, suggested to the writer, a model in which the velocity assignment mechanism is retained, but introducing more general initial distributions. The purpose of this note is to show that in a strong sense the Poisson distribution is the only initial distribution leading to a "stable" traffic flow. Our main result is that for a wide class of initial distributions, the passage of time brings a convergence to the Poisson distribution.

This result was announced at the Congress of the International Statistical Institute, September 1961, [1]. At that time Alan Miller of Birmingham University, and George Weiss of the University of Maryland informed me that they had each independently obtained similar results [2], [3]. After some discussion we established that their proofs held only for initial distributions such that the gaps between points were independent and identically distributed, and that they had both used analytic methods of proof, which differed considerably from our approach. Since the present proof is short and relatively uncomplicated, it may have some merit.

It is a pleasure to acknowledge the pleasant and illuminating conversations on this subject with Frank Haight.

2. The Model. At $t = 0$, let there be a set of starting points $X_1, X_2, \cdots$ on the negative real axis which are obtained as observations of a stochastic point process. Concerning this process we assume

(a) A spatial density σ exists with probability one, i.e.,

$$\lim_{x\to\infty} \{\text{no. of } X_k \text{ in } [0, -x]\}/x = \sigma,$$

with probability one.

(b) There is no "clumping up" of autos, i.e., for any finite interval I, the ex-

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