CONVERGENCE THEOREMS FOR MULTIPLE CHANNEL LOSS PROBABILITIES[1]

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1. Introduction and summary. We are concerned in this paper with a process characterized by the arrival of units at a facility consisting of an integral number c of channels or servers, who are to process these units. Denote by the random variable t_n the arrival time at the facility of the nth unit, and by $T_n = t_n - t_{n-1}$ the inter-arrival time between the $(n-1)$st and nth units. The random variables $\{T_n\}$, $n = 1, 2, \cdots$, are assumed to be independent and identically distributed (I.I.D.) as some typical random variable T with distribution function (d.f.) F_T. The trivial case $F_T(0) = 1$ is excluded. If an arriving unit finds channels free, it is processed by any one of them. The channels behave independently and identically, in the sense that the processing time of a unit does not depend on the particular channel doing the processing, or on the status of the other channels. If an arriving unit finds all channels busy, it departs, or is lost. It is assumed that the nth unit has associated with it a processing time R_n, whether or not it is in fact processed. The $\{R_n\}$, $n = 1, 2, \cdots$ are taken to be random variables which are I.I.D. as some typical R with d.f. F_R, and are furthermore to be independent of the $\{T_n\}$. Unless it is stated to the contrary, we assume that R has finite expectation.

In a typical example, the units might be messages or telephone calls, and the channels be lines or cables over which the messages or calls are transmitted. The $\{T_n\}$ would be the time between attempted calls. If all lines are busy when the nth call is attempted, the call or message is lost. The $\{R_n\}$ represent the length of conversation that would follow from the nth call if that call were to find a free line. Among the quantities which characterize the reliability of such a system are the probability p_n that the nth call or message is not lost, and the probability p_t that at some specified time t not all lines or cables are busy. We shall be concerned here with the convergence of $\{p_n\}$ and $\{p_t\}$ (as $n \to \infty$ and $t \to \infty$), as well as that of a larger class of probabilities.

The question of the convergence of the sequence $\{p_n\}$ was first studied for the case of one channel by F. Pollaczek in the last chapter of his book [2]. Under certain restrictions he proves the convergence by somewhat lengthy, and purely analytical methods. In Section 2 we indicate how a very elementary application of renewal theory yields a somewhat more precise result under less conditions.

It was also Pollaczek who first posed the question of the convergence of $\{p_n\}$

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