SIMPLIFIED ESTIMATES FOR THE EXPONENTIAL DISTRIBUTION[1]

By A. E. Sarhan,[2] B. G. Greenberg, and Junjiro Ogawa[3]

University of North Carolina

1. Introduction and summary. In many practical applications involving statistical estimation, "inefficient" estimates may be the ones of choice for reasons of economy, in money, time, and effort. The usefulness of such estimates, which are generally based upon order statistics, was highlighted by Mosteller [6] who reasoned that observations in large samples could easily be arranged in order of magnitude by punch-card equipment. Moreover, there are many instances where the observations in the sample, by the nature of the measurement, occur naturally in order of magnitude. This happens, for example, in the case of fatigue and life-testing studies. This study deals with certain estimation problems of the one- and two-parameter exponential distributions.

The present study provides, for the parameters of the exponential distribution, estimators that are linear functions of specific subsets of the order statistics. These estimators are optimal in the sense that they provide the most efficient linear combinations of a given number of order statistics.

The use of only two observations from a sample, particularly a small one, represents a situation meriting special study. This paper will first consider which are the best two and which are the worst two order statistics to select for estimation purposes in samples up to size 20. Some consideration will also be given to the use of symmetrically placed order statistics.

For large samples, Ogawa [8], [9] derived optimum spacings to select subsets ranging in size from 1 to 15 for use as linear estimators in the one-parameter exponential distribution. The present paper will consider the same problem where a portion of the sample has been censored at the beginning of the distribution.

2. Estimation using two observations in small samples.

(a) Consider first the two-parameter exponential distribution given by the density

$$f(x) = \begin{cases} (1/\sigma)e^{-(x-\mu)/\sigma}, & \mu \leqq x, \\ 0, & \text{elsewhere.} \end{cases} \tag{2.1}$$

The purpose is to obtain the minimum variance unbiased linear estimates for μ and σ using only two order statistics from a sample of size n. Suppose that these

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[1] Sponsored by U. S. Army Research Office (Durham).

[2] Present address is Alexandria University, Alexandria, Egypt.

[3] Present address is Nihon University, Tokyo, Japan.