

# THE TWO-SAMPLE SCALE PROBLEM WHEN LOCATIONS ARE UNKNOWN<sup>1</sup>

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**1. Introduction.** Let  $X_1, X_2, \dots, X_m$  and  $Y_1, Y_2, \dots, Y_n$  be independent observations from two distributions with cdf  $F(x - \nu)/\sigma$  and  $F(x - \eta)/\tau$  respectively, where  $\nu$  and  $\eta$  are location parameters (for example medians),  $\sigma$  and  $\tau$  are scale parameters and  $F$  is an absolutely continuous distribution function. The problem is to test the hypothesis  $H: \sigma = \tau$  against one- or two-sided alternatives. In addition to the classical  $F$ -test, there are several tests available for this problem (see Klotz [6], [7], Sukhatme [11], [12], Ansari and Bradley [1], Barton and David [2], Siegel and Tukey [10] and Mood [8]). Some of these tests assume the knowledge of the location parameters  $\nu$  and  $\eta$  and some the equality of  $\nu$  and  $\eta$ . For the general problem where the location parameters are completely unknown, the possibility of applying the usual tests to the deviations of the observations from certain consistent estimates of the unknown location parameters has been recognized by several workers in the field. See for example [1]. Sukhatme [12] has constructed a test for this case which is asymptotically distribution-free under certain conditions on the underlying distributions. Crouse [4] has recently shown that the test proposed by Mood [8] when modified in the above manner is asymptotically distribution-free under certain conditions.

In this paper we consider the normal scores test proposed by Klotz [7] for the case  $\nu = \eta$  (unknown) and modify it to apply for the case when the location parameters are completely unknown. The limiting distribution of the modified test statistic is shown to remain unchanged by this modification if  $F$  is symmetric and satisfies certain regularity conditions. It follows that the modified test is asymptotically distribution-free for a fairly general class of distributions. In Section 4 it is shown that the relative asymptotic efficiency of the proposed test and the Studentized  $F$ -test also remains unchanged. In the same section this efficiency has been studied for some standard distributions.

**2. Assumptions and notations.** Let  $X_1, X_2, \dots, X_m$  be independent random variables with common continuous cumulative distribution function  $F(x - \nu)$ . Let  $Y_1, Y_2, \dots, Y_n$  be independent random variables with common continuous cumulative distribution function  $G(x - \eta)$ . We shall assume throughout that the distributions  $F$  and  $G$  have densities  $f$  and  $g$  respectively and that  $\nu$  and  $\eta$  are the medians of  $F$  and  $G$  respectively. Let  $N = m + n$ ;  $\lambda_N = m/N$  and assume

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