# CONSENSUS OF SUBJECTIVE PROBABILITIES: A CONVERGENCE THEOREM[1]

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We investigate here an 'economic' model with $m$ individuals and $n$ objects. We assume that each individual has a certain endowment and places a certain value on each object, and ask whether one can make dynamic assumptions about the behavior of the individuals which will insure that in the course of time 'social' values will be attached to the objects—values which in some sense represent a consensus of the values given them by individuals.

We consider a simple dynamic mechanism and show that in the course of time it leads to unique 'social' values for each object. The dynamic mechanism, although extremely simple, acts as an efficient 'feed-back' mechanism, adjusting the values towards the 'social' values.

The model can be interpreted [3] as an economic exchange model in which the consumers' preferences are given by linear utility functions. Here, however, we interpret it in terms of the type of consensus represented by the pari-mutuel method of betting on horse races. In this system the final 'track's odds' on a given horse are proportional to the amount bet on that horse.

In formulating the pari-mutuel model we assume that the $m$ individuals involved are bettors, labeled $B_1, \cdots, B_m$, concerned with one particular race involving $n$ horses, labeled $H_1, \cdots, H_n$. We assume further that each $B_i$ has arrived at an estimate of the relative merits of each of the $H_j$'s which he expresses in quantitative terms. Specifically, we are given an $m \times n$ *subjective probability matrix* $P = (p_{ij})$ where $p_{ij}$ is the probability, in the opinion of $B_i$, that $H_j$ will win the race. We may as well assume that each column of the matrix $P$ contains at least one positive entry. If this were not so then, say $p_{ij} = 0$ for all $i$, and we could then eliminate $H_j$ from consideration entirely.

Having determined his subjective probability distribution, $B_i$ will now bet the amount $b_i$, a fixed positive number called $B_i$'s *budget*, in a way which maximizes his *subjective expectation*. This means, of course, that $B_i$ will not necessarily bet the whole amount $b_i$ on that $H_j$ for which $p_{ij}$ is largest. In general, $B_i$ will 'bet the odds', that is he will consider the current track odds, or, more conveniently, the current track probabilities. If these are $\pi_1, \cdots, \pi_n$, he will examine the ratios $p_{ij}/\pi_j$ and in some way distribute $b_i$ among those $H_j$ for which this ratio is a maximum. We shall refer to this course of action as *$B_i$'s strategy*. It will be convenient to choose the unit of money so that $\sum_{i=1}^m b_i = 1$.

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