STRINGENT SOLUTIONS TO STATISTICAL DECISION PROBLEMS[1]

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1. Introduction. The statistical decision problem generally involves performing some kind of experiment which yields partial and uncertain information about a parameter or an *a priori* distribution, and decision theory studies the risks resulting from basing action on the partial and uncertain nature of this information. When, implicit in the decision problem, more reliable information is available about some relevant aspects of the parameter or *a priori* distribution than about others, a natural breaking up of the parameter space or space of *a priori* distributions into equivalence classes, or "slices" (see Wesler [13]), is suggested. The equivalence classes are chosen so there is relatively good information about which equivalence class the parameter or *a priori* distribution lies in. In this situation the stringent decision functions appear as a class of conservative solutions to the problem, in the way that the minimax solution appears in the unsliced problem. The notion of a stringent solution has arisen in at least two particular contexts previously: the first in the theory of testing hypotheses, see Lehmann [7] (the proper source is Hunt and Stein [3]), the second in prediction theory, see Lehmann [6].

Section 2 introduces the concept of a stringent solution for the statistical decision problem and develops some elementary aspects of the theory of stringency.

Section 3 applies the concept of stringency to the multivariate (non-Bayesian) problem obtained by repeating a basic decision problem, and to the empirical Bayes problem (see Robbins [10]). These two problems are shown to be closely related from the viewpoint of stringency. The possibility of using stringent or nearly stringent solutions in treating the empirical Bayes problem is discussed, and then it is shown how these solutions may be used in some cases to obtain interesting solutions to the multivariate problem.

Section 4 provides two examples of applications of the ideas developed in Section 3 to estimation problems with squared error loss function. These problems were chosen to reveal a maximum amount of the relevant structure with a minimum of extraneous complication; stringency is in no way restricted to such problems.

So far as I know, the results in Section 3 venture in a hitherto unexplored area. They leave much to be desired, but I hope they may interest someone else in further exploration.

The theory set forth in this paper should be compared with alternative ap-

Received 4 September 1964; revised 26 October 1966.

[1] Prepared with the partial support of the U. S. Army Research Office, DA-ARO(D) 31-124-G-83.