ESTIMABILITY OF VARIANCE COMPONENTS FOR THE TWO-WAY CLASSIFICATION WITH INTERACTION

BY DAVID A. HARVILLE

Aerospace Research Laboratories, Wright-Patterson AFB

1. Introduction and summary. Graybill and Hultquist [2] have defined a variance component to be estimable if there exists a quadratic function of the observations having expectation equal to the component. This definition (extended to functions of variance components) will be used in the present paper in investigating certain aspects of the estimability of linear functions of variance components for the two-way completely-random classification with unequal numbers of observations in the subclasses. It is obvious that at least some functions of the variance components are not estimable for certain sets of subclass numbers.

The objectives underlying the present paper were (1) to derive, for the two-way classification, necessary and sufficient conditions which must be satisfied by the subclass numbers in order for linear functions of the variance components to be estimable and (2) to determine, for the same classification, those sets of subclass numbers for which two commonly-used variance-component estimation procedures, Methods 1 and 3 of Henderson [3], yield unbiased estimates of the components or linear functions of the components.

Observations y_{ijk} are taken as having the linear model

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk},$$

with $i = 1, \cdots, a; j = 1, \cdots, b$; and $k = 1, \cdots, n_{ij}$. μ is a general mean, the α_i and the β_j are main effects, the γ_{ij} are interaction effects, and the ϵ_{ijk} are residual effects. μ is regarded as fixed while the $\alpha_i, \beta_j, \gamma_{ij}$, and ϵ_{ijk} are taken to be mutually-independent random variables with zero means and variances $\sigma_\alpha^2, \sigma_\beta^2, \sigma_\gamma^2$, and σ_ϵ^2. The total number of filled subclasses (subclasses such that $n_{ij} \geqq 1$) will be denoted by p. It is assumed that $a \geqq b$, which can be done without loss of generality.

Methods 1 and 3 (of Henderson) for estimating variance components are based on the analyses of variance given in Tables 1 and 2 respectively where, letting $n_{i\cdot} = \sum_j n_{ij}$, $n_{\cdot j} = \sum_i n_{ij}$, and $n_{\cdot\cdot} = \sum_i n_{i\cdot} = \sum_j n_{\cdot j}$ and using ordinary notation for means, $R_0 = \sum_{ijk} y_{ijk}^2$, $R_\mu = n_{\cdot\cdot}\bar{y}_{\cdot\cdot\cdot}^2$, $R_\alpha = \sum_i n_{i\cdot}\bar{y}_{i\cdot\cdot}^2$, $R_\beta = \sum_j n_{\cdot j}\bar{y}_{\cdot j\cdot}^2$, and $R_\gamma = \sum_{ij} n_{ij}\bar{y}_{ij\cdot}^2$. Also, taking the $b \times 1$ vector $\hat{\boldsymbol{\beta}}$ to be any solution to

$$\mathbf{W}\hat{\boldsymbol{\beta}} = \mathbf{q} \tag{1}$$

where the elements of the $b \times b$ matrix $\mathbf{W}$ are

$$w_{jj} = n_{\cdot j} - \textstyle\sum_i (n_{ij}^2/n_{i\cdot}), \qquad j = 1, \cdots, b,$$

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