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ON A LIMIT DISTRIBUTION OF HIGH LEVEL CROSSINGS OF A STATIONARY GAUSSIAN PROCESS[1]

BY CLIFFORD QUALLS

University of New Mexico

1. Introduction. Let $\{\xi(t), -\infty < t < \infty\}$ be a real stationary Gaussian process with zero mean function and having continuous sample paths with probability one. Denote the covariance function by r (taking $r(0) = 1$ for convenience), and the corresponding spectral distribution function by F. Let μ be the expected number of upcrossings of the level u by $\xi(t)$ in a t-interval of length 1.

H. Cramér [4] in a recent and important paper proved that for a certain class of these processes the number of upcrossings of a level tending to infinity during a t-interval of length T behaves asymptotically like a Poisson process provided the time T is measured in units of $1/\mu$. Cramér's requirements determining this class of processes are:

$$(1') \qquad r^{(iv)}(0) \text{ exists, or equivalently, } \int_{-\infty}^{\infty} \lambda^4 dF(\lambda) < \infty,$$

and

$$(2) \qquad r(t) = O(t^{-\alpha}) \text{ as } t \to \infty \text{ for some } \alpha > 0$$

In this paper we shall show that Cramér's result applies to a significantly wider class of stationary Gaussian processes by replacing his condition $(1')$ by a condition only slightly stronger than the existence of $r''(0)$:

$$(1) \qquad \lambda_2 = -r''(0) \quad \text{exists and} \quad \int_0^{\delta} (\lambda_2 + r''(t))/t \, dt < \infty \quad \text{for some } \delta > 0,$$

$$\text{or equivalently,} \qquad \int_0^{\infty} \log(1 + \lambda)\lambda^2 \, dF(\lambda) < \infty.$$

(The equivalence of the two statements in condition (1) was proved in [9], but now essentially the same proof can be found in a recent paper by R. P. Boas [3], Theorem 3.) To be more precise, the result proved in this paper is the following limit theorem.

THEOREM 1.1. *Suppose the process $\xi(t)$ satisfies conditions* (1) *and* (2). *Let $N(a_i, b_i)$ be the number of upcrossings of the level u by $\xi(t)$ in the t-interval (a_i, b_i). The t-intervals $(a_1, b_1), \cdots, (a_j, b_j)$ are disjoint and depend on the level u in that $b_i - a_i = \tau_i/\mu$, $i = 1, \cdots, j$, where $\tau_1, \cdots, \tau_j$ are fixed positive numbers. Then for every j-tuple of non-negative integers $k_1, \cdots, k_j$,*

$$\lim_{u\to\infty} P\{N(a_i, b_i) = k_i, \ i = 1, \cdots, j\} = \prod_{i=1}^{j} e^{-\tau_i}\tau_i^{k_i}/k_i!.$$

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[1] This paper is based on part of the author's doctoral dissertation completed June 1967 at the University of California at Riverside under the direction of Professor Howard G. Tucker.

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