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B. RAMANCHANDRAN. *Advanced Theory of Characteristic Functions*. Statistical Publishing Society, Calcutta, 1967. vii + 208 pp. \$7.00.

Review by R. CUPPENS

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This book gives an account of the present state of the theory of analytic characteristic functions. Therefore, a better title would be "Theory of analytic characteristic functions". It can be decomposed in three parts: general theory (Chapters 1–3), decomposition theory (Chapters 4–7), applications (Chapter 8).

After a Chapter 0 devoted to some results of the theory of functions of real and complex variables, Chapter 1 recalls (mostly without proof) the elementary theory of characteristic functions: distribution functions, moments, fundamental properties of characteristic functions, infinitely divisible and lattice distributions. [It must be noted that the definition of "negative binomial-type distribution function" (p. 25) is not the classical one.] In the Chapter 2, the author studies analytic characteristic functions and their extensions (defined by J. Marcinkiewicz and studied recently by C. G. Esseen and the author): the characteristic functions which are boundary values of analytic functions. This chapter contains the Raikov's theorem on analyticity strip, its extension to boundary characteristic functions, the relation between boundary characteristic functions and bounded distributions, the Raikov's theorem on decompositions of analytic characteristic functions and the validity in the analyticity strip of the classical (Lévy, Lévy-Khintchine, Kolmogorov) representations for characteristic functions which are infinitely divisible and analytic. [The definition of boundary characteristic functions given here is not the simplest possible: indeed, it is sufficient to say that a characteristic function is boundary value of an analytic function g, that is $\lim_{y\to 0+} g(t + iy) = f(t)$: the continuity of g in the closed strip follows immediately from the uniform continuity of f. Moreover, the proof of Theorem 2.1.2 (the purely imaginary points of the boundary of the strip are singular points) is unusefully intricate: the characteristic function f is analytic in the strip $\{-\alpha < \operatorname{Im} z < \beta\}$ if and only if $\int_{-\infty}^{+\infty} e^{-yx}\, dF(x)$ converges for $-\alpha < y < \beta$. If β is not a singular point, then this integral converges for $-\alpha < y < \beta'$ with $\beta' > \beta$ and f is analytic in the strip $\{-\alpha < \operatorname{Im} z < \beta'\}$; this is in contradiction with the hypothesis and proves the theorem]. In Chapter 3, a detailed account of some results on order and type of entire characteristic functions due to P. Lévy, Pólya and the author is given.

With Chapter 4, begins the study of decomposition theorems. It gives the classical theorems on the decomposition of normal, Poisson and binomial laws and especially the Linnik theorem (with the simple proof due to I. V. Ostrovskiy) on the decomposition of the convolution of normal and Poisson laws. Chapter 5 studies the unsolved problem of the characterization of the class I_0 of infinitely divisible characteristic functions without indecomposable factors.