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THE ASYMPTOTIC BEHAVIOR OF BAYES' ESTIMATORS[1]

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1. Introduction. Schwartz [9] wrote in 1964 that "until quite recently, very little was published with regard to the asymptotic behavior of Bayes' procedure." The phrase "very little", in the author's opinion, is with respect to the amount of literature concerning maximum likelihood estimator (MLE). In fact, many outstanding works have already been done by various authors. Doob [4], by using a martingale argument and under very weak conditions, established the consistency of Bayes' estimator for almost all parameter points. LeÇam [5], [6], under a set of conditions which is stronger than those for consistency of the MLE, proved the consistency of Bayes' estimator for every parameter point.

It remained for Schwartz [9], [10] to take the major steps. Initiated by Blackwell and stimulated by LeCam, the result she presented can be roughly stated as follows. The Bayes' estimator is consistent if there exists a consistent estimator.

The purpose of this memorandum is to establish some of the asymptotic properties of Bayes' estimators by showing that the MLE and the Bayes' estimator are asymptotically equivalent. This fact was noticed and informally established by Wolfowitz [11] and Lindley [7]. A complete argument for the case of estimating a one-dimensional parameter was given by Bickel and Yahav [2]. The present memorandum is an extension of their works, the same result for the case of estimating an h-dimensional ($h \geqq 1$) parameter will be proved.

Our result can be used to compute the asymptotic Bayes' posterior risk for the point estimation situation. Once the posterior risk can be computed, it is well known [2], [3] how to find the asymptotically optimal stopping times for the usual sequential setup of the estimation problem.

2. The main theorem. In this section, it is shown that for the point estimation situation, the Bayes' estimator θ_n and the maximum likelihood estimator $\hat{\theta}_n$ are asymptotically equivalent, namely

$$n^{\frac{1}{2}}[\theta_n - \hat{\theta}_n] \to \mathbf{0} \tag{2.1}$$

a.s. P_{θ_0} for all $\theta_0 \in \Theta$.

A direct consequence of (2.1) is that all the asymptotic properties of MLE also hold for the Bayes' estimators. Also, since the determination of the MLE is independent of the loss function and the prior measure, the asymptotic properties of Bayes' estimator hold for all priors and loss functions in a certain class. This can

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