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ON THE DOMAINS OF DEFINITION OF ANALYTIC CHARACTERISTIC FUNCTIONS[1]

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1. Introduction. It is known that analytic characteristic functions are always regular in a horizontal strip (the strip of regularity) which contains the real axis in its interior. It is, however, often possible to continue an analytic characteristic function beyond its strip of regularity. The problem arises whether an analytic characteristic function can have a natural domain of analyticity, that is, a domain beyond which it cannot be continued analytically. The accessible points of the boundary of this domain are then singular points. I. V. Ostrovskii [4] used a theorem of Cartan and Thullen (see Bochner–Martin [1] page 84) to show that any domain which is symmetric with respect to the imaginary axis and which contains a horizontal strip and the real axis can be the domain of analyticity of a characteristic function.

In the present note we construct by an elementary method characteristic functions which have a given domain of analyticity. The results are then extended to characteristic functions which are boundary functions of analytic functions.

2. Some lemmas.

LEMMA 1. *The function*

$$f(t)=\left[\left(1+\frac{it}{\alpha}\right)\left(1+\frac{it}{\beta+i\gamma}\right)\left(1+\frac{it}{\beta-i\gamma}\right)\right]^{-1}$$

where α, β and γ are real numbers is a characteristic function if $0<\alpha\leqq\beta$.

The lemma is proven by expanding $f(t)$ into partial fractions and applying the inversion formula.

LEMMA 2. *It is always possible to find a sequence $\{a_k\}$ of positive numbers such that $\sum_{k=1}^{\infty}a_k=1$ while $a_n\geqq 2\sum_{k=n+1}^{\infty}a_k$. An example of such a sequence is $a_k=(c-1)c^{-k}$ $(k=1,2,\cdots)$ with $c>3$.*

LEMMA 3. *Let G be an arbitrary domain; then there exists a denumerable set of points which is dense in the boundary ∂G of G.*

We consider the set of all points of G which have rational coordinates and arrange them in a sequence $\{\alpha_n\}$. For each point α_n let β_n be the point of ∂G which is nearest to α_n and which has the property that $\arg(\alpha_n-\beta_n)$ has the smallest possible value. In general the sequence $\{\beta_n\}$ contains repetitions. We omit these and obtain a sequence $\{b_n\}$ which is dense in ∂G.

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