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A MULTI-PARAMETER GAUSSIAN PROCESS[1]

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1. Introduction. Let $(\Omega, \mathscr{F}, P)$ be a probability space and let A be the p-dimensional unit rectangle $(p \geqq 2)$. We denote by $(A, \mathscr{A}, \mu_p)$ the ordinary Lebesgue measure space. Let $\{X(u, \omega): u \in A\}$ be a Gaussian process defined on $(\Omega, \mathscr{F}, P)$ with the properties:

$$X(u, \omega) = 0 \quad \text{a.s.} \quad \text{for every} \quad u \quad \text{in} \quad A_0 \quad \text{where} \tag{1.1}$$

$$A_0 = \{(u_1, \cdots, u_p) \in A : u_j = 0 \ \text{for some} \ j \ \text{with} \ 1 \leqq j \leqq p\}.$$

$$E[X(u, \omega)] = \textstyle\int_\Omega X(u, \omega)\, dP(\omega) = 0 \quad \text{for every} \quad u \quad \text{in} \quad A. \tag{1.2}$$

$$E[X(u, \omega)X(v, \omega)] = \min(u_1, v_1) \cdots \min(u_p, v_p) = R(u, v) \tag{1.3}$$

$$\text{for every} \quad u = (u_1, \cdots, u_p) \quad \text{and} \quad v = (v_1, \cdots, v_p) \quad \text{in} \quad A.$$

By considering an expansion in terms of Haar functions on A, it is shown that $X(u, \omega)$ can be realized in the space $C(A)$ of real continuous functions on A which vanish at A_0, i.e.

(1.4) Almost all sample functions of $X(u, \omega)$ are continuous.

For $p = 2$, the existence of the above Gaussian process $X(u, \omega)$ is shown by Yeh [15] and Kuelbs [10]. We will call a Gaussian process $X(u, \omega)$ with the properties (1.1)–(1.4) the p-parameter Gaussian process. We then examine the interrelationship between the p-parameter Gaussian process and its reproducing kernel Hilbert space $H(R)$. Let $L^2(A)$ denote the space of Lebesgue square-integrable functions on A with an inner product $(f, g) = \int_A f(u)g(u)\, d\mu_p(u)$ and norm $\| \ \|$. We also define a stochastic integral $I(f) = \int_A f(u)\, dX(u, \omega)$ for $f \in L^2(A)$ with respect to the p-parameter Gaussian process in two different ways and show that they are identical. From this we show that the p-parameter Gaussian process has an a.s. uniformly convergent orthonormal expansion.

Defining a Gaussian random set function by

$$X(F, \omega) = \textstyle\int_A 1_F(t)\, dX(t, \omega) \tag{1.5}$$

where $F \in \mathscr{A}$ and 1_F is the indicator function of F, we define the multiple Wiener integral (see Itô [6]) and show that any L^2-functional of the process has an orthogonal representation.

By appealing to the results obtained by Parzen [13], Kallianpur [9] and Oodaira [11], we can simply deduce the results: a translation theorem, equivalence of

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