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ABSTRACTS

(*Abstract of a paper presented at the Eastern Regional meeting, Chapel Hill, North Carolina, May* 5–7, 1970. *Additional abstracts appeared in earlier issues.*)

125-54. Some problems in regression design and approximation theory. GRACE WAHBA, University of Wisconsin. (Invited)

Let $X(t)$, $t \in [0, 1]$, be a zero mean stochastic process with $EX(s)X(t) = R(s, t)$ and let $Y(t) = \theta f(t) + X(t)$, $t \in [0, 1]$, where θ is a parameter to be estimated and f is known. Suppose $Y(t)$ possess $m-1$ quadratic mean derivatives. For each n, it is desired to choose a design $T_{n^1} T_n = \{t_{0n} < t_{1n^1} < \cdots, t_{nn}\}$ so that the Gauss–Markov estimate for θ, based on $Y^{(\nu)}(t_{in})$, $\nu = 0, 1, \cdots, m-1$, $i = 0, 1, \cdots, n$, is as small as possible. Let $X(t)$ formally satisfy the stochastic differential equation $L_m X(t) = dW(t)/dt$, $X^{(\nu)}(0) = \xi_\nu$, $\nu = 0, 1, \cdots, m-1$, where $W(t)$ is a Wiener process and the $\{\xi_\nu\}$ are normal, zero mean random variables, independent of $W(t)$. L_m is an mth order linear differential operator whose null space is spanned by an extended, complete, Tchebychev system of continuity class C^{2m}. Let $f(t) = \int_0^1 R(t, u)\rho(u)\, du$, where ρ is strictly positive and possess a bounded first derivative. Let $\lim_{s \downarrow t} \partial^{2m-1}/\partial s^{2m-1} R(s, t) - \lim_{s \uparrow t} \partial^{2m-1}/\partial s^{2m-1} R(s, t) = (-1)^m \alpha(t)$. Then $T_n^* = \{t_{0n}^*, t_{1n}^*, \cdots, t_{nn}^*\}$, $n = 1, 2, \cdots$ with t_{in}^* given by $t_{0n}^* = 0$, $\int_0^{t_{in}^*} [\rho^2(u)\alpha(u)]^{(2m+1)^{-1}}\, du = i/n \int_0^1 [\rho^2(u)\alpha(u)]^{(2m+1)^{-1}}\, du$, $i = 1, 2, \cdots, n$, is an asymptotically optimum sequence of designs and $(1/\sigma^2 - 1/\sigma_{T_n^*}^2) = [(m!)^2/n^{2m}(2m)!(2m+1)!] \times [\int_0^1 [\rho^2(u)\alpha(u)]^{(2m+1)^{-1}}\, du] + o(1/n^{2m})$, where σ^2 and $\sigma_{T_n^*}^2$ are the variances of the Gauss–Markov estimates of θ based on $Y(t)$, $t \in [0, 1]$, and $Y^{(\nu)}(t)$, $\nu = 0, 1, 2, \cdots m-1$, $t \in T_n^*$, respectively. This extends some results of Sacks and Ylvisaker, [*Ann. Math. Statist.* **37**, 66–89; **39**, 49–69; and unpublished manuscript], whose definition of asymptotically optimum we use. This regression design problem is shown to be equivalent to some problems in approximation theory, in particular the establishment of optimal quadrature formulae of a certain type. (Received October 7, 1970.)

(*Abstracts of papers to be presented at the Eastern Regional meeting, University Park, Pennsylvania, April* 21–23, 1971. *Additional abstracts will appear in future issues.*)

129-1. Construction of maximal designs of resolution VII and VIII. BODH RAJ GULATI, Southern Connecticut State College.

A symmetrical factorial design based on N runs, k factors each operating at two levels, is said to be of resolution t if no main effect or $(t-1)$-factor $(t > 2)$ or its lower order interaction is confounded with block effects. Such a design may symbolically be denoted by (N, k, t)-design. An (N, k, t)-design is said to be *maximal* if there exists no other (N, k^*, t)-design with $k^* > k$. This paper produces the following maximal resolution VII designs: (i) (64, 7, 7), (ii) (128, 8, 7), (iii) (192, 7, 7), (iv) (256, 9, 7), (v) (320, 7, 7), (vi) (384, 8, 7), (vii) (512, 11, 7), (viii) (1024, 15, 7) and (ix) (2048, 23, 7). Maximal designs of resolution VIII follow immediately, since it is known that if (N, k, t)-design with largest k and $t = 2u+1$ exists, so does $(2N, k+1, t+1)$. (Received August 10, 1970.)

129-2. On the sign test for symmetry. JOSEPH L. Gastwirth, The Johns Hopkins University.

The sign test of the null hypothesis that n observations are from a density which is symmetric about a specified value μ is based on the number of observations which are less than μ and rejects the symmetry hypothesis if this number is too far from its expected value $(n/2)$. In practice one does not know μ and might estimate it by the sample mean $(\bar{X})$. The modified sign test counts the number of observations which are less than $\bar{X}$. The asymptotic distribution of the modified sign

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