The Annals of Mathematical Statistics
1971, Vol. 42, No. 3, 1079-1083

A BOUND ON TAIL PROBABILITIES FOR QUADRATIC FORMS IN INDEPENDENT RANDOM VARIABLES[1]

BY D. L. HANSON AND F. T. WRIGHT[2]

University of Missouri and University of Iowa

Let X_i for $i = 0, \pm 1, \cdots$ be independent random variables whose distributions are symmetric about zero and such that

$$P\{|X_i| \geqq x\} \leqq M \textstyle\int_x^\infty e^{-\gamma t^2}\, dt \tag{1}$$

for all i and all $x \geqq 0$ where M and γ are positive constants. Suppose a_{ij} for $i, j = 0, \pm 1, \cdots$ are real numbers such that $a_{ij} = a_{ji}$ for all i and j, and such that

$$\Lambda^2 = \textstyle\sum_{i,j} a_{ij}^2 < \infty. \tag{2}$$

Let A denote the matrix $((|a_{ij}|))_{i,j=0,\pm 1,\cdots}$, and let $\|A\|$ be the norm of A considered as an operator on l_2, the index on the sequences in l_2 taking on the values $0, \pm 1, \cdots$. Define

$$S = \textstyle\sum_{i,j} a_{ij}(X_i X_j - EX_i X_j). \tag{3}$$

The purpose of this paper is to prove:

THEOREM. *Under the assumptions stated above, S exists as a limit, both in quadratic mean and almost surely, of the sequence*

$$\{S_N = \textstyle\sum_{i,j=-N}^{N} a_{ij}(X_i X_j - EX_i X_j)\},$$

and there exist constants C_1 and C_2 depending on M and γ (but not on the coefficients a_{ij}) such that for every $\varepsilon > 0$

$$P\{S \geqq \varepsilon\} \leqq \exp(-\min\{C_1 \varepsilon / \|A\|, C_2 \varepsilon^2 / \Lambda^2\}). \tag{4}$$

If $\{Y_k = \sum_v \alpha_v X_{k-v}\}$ is a moving average, then quadratic sums of the form (3) occur naturally when estimating its spectral density. This was the original motivation behind our work.

We would very much like to remove the restriction that the distributions of the X's be symmetric. Unfortunately, our proof depends heavily on this symmetry.

Received August 17, 1970.

[1] Research partially sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under Grant No. AFOSR-68-1394.

[2] Part of this research was done while the author held a summer fellowship from the Old Gold Foundation at the University of Iowa.