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ON A CLASS OF BIVARIATE DISTRIBUTIONS FOLLOWING A CERTAIN STOCHASTIC STRUCTURE[1]

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0. Introduction. Consider two independent random variables X, Y with $E(X) = E(Y) = 0$ and two other random variables X^*, Y^* following a Stochastic Structure:

$$X^* = AX+BY; \tag{1}$$

$$Y^* = CX+DY;$$

where A, B, C, D are nonzero constants.

Laha ([2], [4]) studied the problem of characterizing the distributions of X and Y through regression properties of X^* and Y^*. In particular, he showed that if $AD = BC$, $E(Y^* \mid X^*) = \beta X^*$ almost surely whatever may be the distributions of X and Y, where $\beta = DB^{-1}$. If $AD \neq BC$, both X and Y have symmetric stable distributions with the same characteristic exponent $\alpha(1 < \alpha \leqq 2)$, if and only if

(i) $E(Y^* \mid X^*) = \beta X^*$ for all $0 < |A| \leqq \delta$ for some $\delta > 0$ and

(ii) $$\beta = (CA^{-1}\alpha_1 |A|^\alpha + DB^{-1}\alpha_2 |B|^\alpha)$$

$$(\alpha_1 |A|^\alpha + \alpha_2 |B|^\alpha)^{-1}; \tag{2}$$

where α_1 and α_2 are the scale parameters of the distributions of X and Y respectively.

The object of this article is to make some extensions of these results. In Section 1, we consider a stochastic structure similar to (1) when $\mathbf{X}$ is a $p \times 1$ random vector, $\mathbf{Y}$ is a $q \times 1$ random vector, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are matrices of order $q \times p$, $q \times q$, $r \times p$, $r \times q$ respectively. Assuming the matrix $\mathbf{B}$ to be nonsingular we show that if $\mathbf{C} = \mathbf{DB}^{-1}\mathbf{A}$, $E(\mathbf{Y}^* \mid \mathbf{X}^*) = \boldsymbol{\beta}\mathbf{X}^*$ almost surely where $\boldsymbol{\beta} = \mathbf{DB}^{-1}$. In Section 2, we confine ourselves to the case when $p = q = 2$. In this case, if $\mathbf{C} \neq \mathbf{DB}^{-1}\mathbf{A}$ and some additional conditions are satisfied, it is possible to characterize a class of bivariate distributions through the regression properties of $\mathbf{X}^*$ and $\mathbf{Y}^*$. These distributions are not necessarily stable as defined by Lévy ([3] Section 63). In Section 3, we consider a special case of the class of bivariate distributions introduced in the previous section. The latter class is stable and includes bivariate normal distribution.

1. Some preliminary results. Suppose, $\mathbf{X}$ and $\mathbf{Y}$ are random vectors of order $p \times 1$ and $q \times 1$ respectively such that $E(\mathbf{X})$ and $E(\mathbf{Y})$ exist. $\varphi(\mathbf{U}, \mathbf{V})$ is the joint

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