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Rejoinder: "On the Birnbaum Argument for the Strong Likelihood Principle"

Deborah G. Mayo

1. INTRODUCTION

I am honored and grateful to have so many interesting and challenging comments on my paper. I want to thank the discussants for their willingness to jump back into the thorny quagmire of Birnbaum's argument. To a question raised in the paper "Does it matter?", these discussions show the answer is yes. The enlightening connections to contemporary projects are especially valuable in galvanizing future efforts to address foundational issues in statistics.

As long-standing as Birnbaum's result has been, Birnbaum himself went through dramatic shifts in a short period of time following his famous (1962) result. More than of historical interest, these shifts provide a unique perspective on the current problem. Already in the rejoinder to Birnbaum (1962), he is worried about criticisms (by Pratt, 1962) pertaining to applying WCP to his constructed mathematical mixtures (what I call Birnbaumization), and hints at replacing WCP with another principle (Irrelevant Censoring). Then there is a gap until around 1968 at which point Birnbaum declares the SLP plausible "only in the simplest case, where the parameter space has but two" predesignated points [Birnbaum (1968), page 301]. He tells us in Birnbaum (1970a, page 1033) that he has pursued the matter thoroughly, leading to "rejection of both the likelihood concept and various proposed formalizations of prior information." The basis for this shift is that the SLP permits interpretations that "can be seriously misleading with high probability" [Birnbaum (1968), page 301]. He puts forward the "confidence concept" (Conf) which takes from the Neyman–Pearson (N–P) approach "techniques for systematically appraising and bounding the probabilities (under respective hypotheses) of seriously misleading interpretations of data" while supplying it an evidential interpretation [Birnbaum (1970a), page 1033]. Given the many different associations with "confidence," I use (Conf) in this Rejoinder to refer to Birnbaum's idea. Many of the ingenious examples of the incompatibilities of SLP and (Conf) are traceable back to Birnbaum, optional stopping being just one [see Birnbaum (1969)]. A bibliography of Birnbaum's work is Giere (1977). Before his untimely death (at 53), Birnbaum denies the SLP even counts as a principle of evidence (in Birnbaum, 1977). He thought it anomalous that (Conf) lacked an explicit evidential interpretation, even though, at an intuitive level, he saw it as the "one rock in a shifting scene" in statistical thinking and practice [Birnbaum (1970a), page 1033]. I return to this in Section 4 of this rejoinder.

Deborah G. Mayo is Professor of Philosophy, Department of Philosophy, Virginia Tech, 235 Major Williams Hall, Blacksburg, Virginia 24061, USA (e-mail: mayod@vt.edu).

2. BJØRNSTAD, DAWID AND EVANS

Let me begin by answering the central criticisms that, if correct, would be obstacles to what I purport to have shown in my paper. It is entirely understandable that leading voices in a long-lived controversy would assume that all of the twists and turns, avenues and roadways, have already been visited, and that no new flaw in the argument could enter to shake up the debate. I say to the reader that the surest sign that the issue is unsettled is that my critics disagree among themselves about the puzzle and even the key principles under discussion: the WCP, and in one case, the SLP itself. To remind us [Section 2.2]:

> SLP: For any two experiments E_1 and E_2 with different probability models f_1, f_2 but with the same unknown parameter θ, if outcomes $\mathbf{x}^*$ and $\mathbf{y}^*$ (from E_1 and E_2, resp.) determine the same likelihood function $[f_1(\mathbf{x}^*;\theta) = cf_2(\mathbf{y}^*;\theta)$ for all $\theta]$, then $\mathbf{x}^*$ and $\mathbf{y}^*$ should be inferentially equivalent for any inference concerning parameter θ.

A shorthand for the entire antecedent is that $(E_1, \mathbf{x}^*)$ is an *SLP pair* with $(E_2, \mathbf{y}^*)$, or just $\mathbf{x}^*$ and $\mathbf{y}^*$ form an SLP pair (from $\{E_1, \mathrm{E}_2\}$). Assuming all the SLP stipulations, we have

> SLP: If $(E_1, \mathbf{x}^*)$ and $(E_2, \mathbf{y}^*)$ form an SLP pair, then $\mathrm{Infr}_{E_1}[\mathbf{x}^*] = \mathrm{Infr}_{E_2}[\mathbf{y}^*]$.