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Comment: A Selective Overview of Nonparametric Methods in Financial Econometrics

Michael Sørensen

1. INTRODUCTION

Professor Fan should be congratulated for his review that convincingly demonstrates the usefulness of nonparametric techniques to financial econometric problems. He is mainly concerned with financial models given by stochastic differential equations, that is, diffusion processes. I will therefore complement his selective review by discussing some important problems and useful methods for diffusion models that he has not covered. My concern will mainly, but not solely, be with parametric techniques. A recent comprehensive survey of parametric inference for discretely sampled diffusion models can be found in [19].

2. GAUSSIAN LIKELIHOOD FUNCTIONS

In his brief review of parametric methods, Professor Fan mentions the Gaussian approximate likelihood function based on the Euler scheme and states that this method has some bias when the time between observations Δ is large. This is actually a very serious problem. As an example, consider a model with a linear drift of the form $\mu(x) = -\beta(x - \alpha)$ $(\beta > 0)$. The estimator $\hat{\beta}_n$ of β obtained from the Gaussian approximate likelihood based on the Euler scheme converges to

$$(1 - e^{-\beta_0 \Delta})\Delta^{-1}$$

as the number of observations n tends to infinity. Here β_0 denotes the true parameter value. The limiting value of $\Delta\hat{\beta}_n$ is always smaller than one, and the limit of $\hat{\beta}_n$ is always smaller than Δ^{-1}. Thus the asymptotic bias can be huge if Δ is large. A simulation study in [3] demonstrates that also for finite sample sizes an enormous bias can occur. When $\Delta\beta_0$ is small so that $(1 - e^{-\beta_0\Delta})\Delta^{-1} \approx \beta_0$, the asymptotic bias is negligible. The problem is, however, that if we use the approximate likelihood function based on the Euler scheme, there is no way we can know whether $\Delta\beta_0$ is small or large because $\Delta\hat{\beta}_n$ will always tend to be small. I suspect that the nonparametric methods outlined in Sections 3.2 and 3.5 might suffer from a similar shortcoming as they are based on the same type of approximation as the Euler scheme.

A simple solution to this problem is to use an approximate likelihood function where the transition density is replaced by a normal distribution with mean equal to the exact conditional expectation $F(x, \theta) = E_\theta(X_\Delta | X_0 = x)$ and with the variance equal to the exact conditional variance $\Phi(x; \theta) = \mathrm{Var}_\theta(X_\Delta | X_0 = x)$. Here θ is the (typically multivariate) parameter to be estimated. This approach is exactly the same as using quadratic martingale estimating functions; see [3] and [20]. The estimators obtained from quadratic martingale estimating functions have the same nice properties for high frequency observations (small Δ) as the estimators based on the Euler likelihood, but they are consistent for any value of Δ and can thus be used whether or not Δ is small. In most cases there is no explicit expression for the functions $F(x, \theta)$ and $\Phi(x; \theta)$, so often they must be determined by simulation. This requires, however, only a modest amount of computation and is not a problem in practice. If a completely explicit likelihood is preferred, one can approximate $F(x, \theta)$ and $\Phi(x; \theta)$ by expansions of a higher order than those used in the Euler scheme; see [16].

The nonparametric method in Section 3.5 could probably be improved in a similar way by using in (27) and (28) the functions $F(x, \theta)$ and $\Phi(x; \theta)$ (or the higher-order expansions in [16]) instead of the first-order approximation used in the Euler scheme.

3. MARTINGALE ESTIMATING FUNCTIONS

More generally, martingale estimating functions provide a simple and versatile technique for estimation in discretely sampled parametric stochastic differential equation models that works whether or not Δ is small.

Michael Sørensen is Professor and Head, Department of Applied Mathematics and Statistics, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark (e-mail: michael@math.ku.dk).