different model. I think that intrinsic noise with $\sigma_e^2 > 0$ leads us immediately to a stochastic world, and if $\sigma_e^2 = 0$ but $\sigma_\varepsilon^2 \neq 0$ and not small, as is often the case in economics, distinguishing between iid and low-dimensional white chaos will be extremely difficult.

This leads to the question of whether the real world, such as an actual economy, contains chaos. Chatterjee and Yilmaz take the position that it is ubiquitous, finding examples in "such diverse fields as physiology, geology, . . . , economics . . ." and "theoretical models of population biology." There are also theoretical models in economics that produce chaos, but that does not imply that it occurs in practice. I would prefer to suggest the opposite view that there is *no* evidence of chaos outside of laboratories. My reason is that there exists no statistical test, that I know of, that has chaos as its null hypothesis. There are plenty of tests that have as a null H_0:iid (or linear) and are designed to have power against chaos. However, as is well known by all statisticians, if one rejects the null a specific alternative hypothesis cannot be accepted. If a null of linearity or iid is rejected, one cannot accept (white) chaos, as nonlinear stochastic models are also appropriate. For example, the test (based on the correlation dimension) by Brock, Dechert and Scheinkman (1987) (the BDS test) that was applied in Brock and Sayers (1988) often finds evidence of nonlinearity but not of chaos. Until a property P can be found that holds *only* for chaos and not for stochastic series, and a test is based on P with chaos as the null, can there be a suggestion that chaos is found in the real world.

Finally, I would suggest that bifurcation and fractional integrated models are irrelevant for the main topic discussed in the articles, but space limitations prevent me from expanding on this point.

In conclusion, I think that scientists working on the area of chaos are doing a disservice to this important area of research by overselling its relevance, by trying to equate it with randomness and by using concepts (such as probability) that are unnecessary and only lead to confusion. The techniques being developed for analysis of chaotic processes, such as the BDS test or estimates of the Lyapunov exponent, or methods of forecasting using $\sigma_e^2 = 0$, are potentially powerful and useful when applied to truly stochastic, real-world series. There is a need for statistical methods to investigate the properties of these techniques in this case, and this, in my opinion, is the natural link between chaos and statistics.

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Comment: Randomness in Complex Systems

David Griffeath

1. WHAT IS RANDOM?

Professors Berliner, Chatterjee and Yilmaz are to be commended for their thoughtful overviews of the recent explosion in experimental and theoretical research on chaos. They identify a host of challenging statistical questions fundamental to the subject and make timely appeals for the readership of *Statistical Science* to join the fray. Over the past decade, I have tried to track the major currents of chaos, studying many of the articles and books mentioned in the authors' fine reference list. I strongly urge others to peruse those sources and seek out a few.

Berliner and Chatterjee and Yilmaz note that the term "chaos" is not used in a consistent manner by the scientific community; for example, there is no universally accepted mathematical definition. In my experience, the word means so many different things to different people that it threatens to become scientifically dangerous. Apparently, Bernoulli shift, the most basic stochastic process, is deemed chaotic. But how is it distinguished from those strange attractors, delicately perched on the boundary between order and randomness, that have dramatically captured the imagination of both scientists and the general public? The phenomenology of chaos is leaving its mark across a broad spectrum of contemporary culture: from physics to philosophy to recreational computing to textile design. At the hairdresser, I discovered an article in a summer issue of *Gentleman's Quarterly* linking mathematical chaos, Silicon Valley nerds and late-

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