review of multivariate methods in applications. In *Statistics: An Appraisal* (H. A. David and H. T. David, eds.) 309–337. Iowa State Univ. Press.

HARSHMAN, R. A. (1970). Foundations of the PARAFAC procedure: Models and conditions for an "exploratory" multi-mode factor analysis. *UCLA Working Papers in Phonetics* **16** 1–84.

INGLE, JR., J. and CROUCH, S. R. (1988). *Spectrochemical Analysis.* Prentice Hall, Englewood Cliffs, NJ.

KAPTEYN, A., NEUDECKER, H. and WANSBEEK, T. (1986). An approach to n-mode components analysis. *Psychometrika* **51** 269–275.

KROONENBERG, P. M. (1983). Annotated bibliography of three-mode data by means of alternating least-squares algorithms. *British J. Math. Statist. Psych.* **36** 81–113.

KROONENBERG, P. M. (1991). The three-mode side of the three-way world. Technical report, Dept. Education, Leiden Univ.

KRUSKAL, J. B. (1977). Three-way arrays: Rank and uniqueness of trilinear decompositions with application to arithmetic complexity and statistics. *Linear Algebra Appl.* **18** 95–138.

LASTOVICKA, J. L. (1981). The extension of component analyses to 4-mode factor analysis. *Psychometrika* **46** 47–57.

LEE, C.-H. (1988). Multilinear analysis of fluorescence spectra of photosynthetic systems. Ph.D. dissertation, Ohio State Univ.

LEE, C.-H., KIM, K. and ROSS, R. T. (1991). Trilinear analysis for the resolution of overlapping fluorescence spectra. *Korean Biochemistry Journal* **24** 374–379.

LEURGANS, S. E. (1991). Multilinear models: Array formulas. Technical Report 473, Dept. Statistics, Ohio State Univ.

LEURGANS, S. E., ROSS, R. T. and ABEL, R. B. (1992). A decomposition for 3-way arrays. *SIAM J. Matrix. Anal. Appl.* To appear.

MALINOWSKI, E. R. and HOWERY, D. G. (1980). *Factor Analysis in Chemistry.* Wiley, New York.

NDOU, T. T. and WARNER, I. M. (1991). Applications of multidimensional absorption and luminescence spectroscopies in analytical chemistry. *Chemical Reviews* **91** 493–507.

PRINGLE, R. M. and RAYNER, A. A. (1971). *Generalized Inverse Matrices with Applications to Statistics.* Hafner, New York.

RAMOS, L. S., BEEBE, K. R., CAREY, W. P., SANCHEZ, E., ERICKSON, B. C., WILSON, B. E., WANGEN, L. E. and KOWALSKI, B. R. (1986). Chemometrics. *Analytical Chemistry* **58** 294R–315R.

ROSS, R. T., LEE, C.-H., DAVIS, C. M., FAYYAD, E. A. and LEURGANS, S. E. (1991). Resolution of fluorescence spectra of plant pigment-complexes using trilinear models. *Biochimica et Biophysica Acta* **1056** 317–320.

RUSSELL, M. D. and GOUTERMAN, M. (1988a). Excitation-emission-lifetime analysis of multicomponent systems – I. Principal component factor analysis. *Spectrochimica Acta* **44A** 857–861.

RUSSELL, M. D. and GOUTERMAN, M. (1988b). Excitation-emission-lifetime analysis of multicomponent systems – II. Synthetic model data. *Spectrochimica Acta* **44A** 863–872.

RUSSELL, M. D., GOUTERMAN, M. and VAN ZEE, J. A. (1988). Excitation-emission-lifetime analysis of multicomponent systems – III. Platinum, palladium and rhodium porphyrins. *Spectrochimica Acta* **44A** 873–882.

SANCHEZ, E. and KOWALSKI, B. R. (1988a). Tensorial calibration: I. First-order calculations. *Journal of Chemometrics* **2** 247–263.

SANCHEZ, E. and KOWALSKI, B. R. (1988b). Tensorial calibration: II. Second-order calibration. *Journal of Chemometrics* **2** 265–290.

SANCHEZ, E. J. and KOWALSKI, B. R. (1990). Tensorial resolution: A direct trilinear decomposition. *Journal of Chemometrics* **4** 29–45.

SANDS, R. and YOUNG, F. W. (1980). Component models for three-way data: An alternating least squares algorithm with optimal scaling features. *Psychometrika* **45** 39–67.

THAMPI, S. (1991). Resolution of intrinsic protein fluorescence using multilinear analysis. Master's thesis, Ohio State Univ.

TUCKER, L. R. (1963). Implications of factor analysis of three-way matrices for measurement of change. In *Problems in Measuring Change* (C. W. Harris, ed.) 122–137. Univ. Wisconsin Press.

Comment

Jan deLeeuw

INTRODUCTION

Complicated multivariate models, and certainly the models used in multidimensional scaling, are most often used for exploratory purposes. The paper by Leurgans and Ross covers one of the fortunate, but rather exceptional, situations in which we can derive the form of the model from prior scientific knowledge. Another, similar, situation is the conformation of molecules using scaling techniques, and the seriation of artifacts in time or of genes along a chromosome. In this class of

Jan deLeeuw is Professor of Mathematics and Psychology, Director of the Social Statistics Program, Department of Mathematics, University of California, Los Angeles, 405 Hilgard Avenue, Los Angeles, California 90024-1555. e-mail: deleeuw@laplace.sscnet.ucla.edu.

applications, the physical information we have tells us that a multilinear model is appropriate – only the coefficients (mixtures) and dimensionality (number of components) are unknown and must be estimated.

OTHER AREAS

Leurgans and Ross discuss the multilinear models pretty much in the context in which they use them. Thus, it looks a bit as if these models were created for spectroscopy. This is perfectly appropriate in a paper such as this, which concentrates on a particular field of application. But to give a somewhat wider perspective, I'll list a number of other areas, both mathematical and nonmathematical, in which multilinear models have been studied or applied.

1. *Efficient computation of matrix products and*