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Comment

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G. Alastair Young's essay states as its theme: "We will discuss reasons why, though a theoretical success, the bootstrap may be judged to have been a less spectacular success in recent years than many might have expected or than should be possible." Expectations are a personal matter, not widely shared. Young's specific concerns include the following:

(a) "...bootstrap procedures which have been developed to handle more complex problems, such as those involving dependent data, are generally not automatic in that they require choice of some form of design parameter" (Section 3).
(b) "Patch-ups of the basic bootstrap involving devices such as modification of resampling size, while understood theoretically, suffer still from a lack of practicality" (Section 4.2).
(c) "Published applications of the bootstrap are now numerous..." but the latest discoveries of bootstrap theory have not made their way into such data analyses (Section 3).
(d) "Researchers have succumbed too much, perhaps, to the temptation to devote their efforts to squeezing even better performance from the bootstrap...rather than focusing their efforts on more fundamental issues concerning basic reliability of the approach" (Section 3).
(e) "Schenker (1985) illustrates the poor small-sample performance of procedures, which have asymptotic justification, when constructing [bootstrap] confidence intervals for a population variance" (Section 3). "Only recently has attention been paid to the practically crucial question of providing the user with some means of assessing how well-determined, or accurate, the bootstrap estimator is" (Section 4.2).
(f) "...there is still much theoretical analysis of bootstrap required before we can be confident of its value. Second, there is need for readily accessible software" (Section 6).
(g) "The very term 'bootstrap,' rightly or wrongly, evokes qualms with many, as producing something out of nothing. Many will feel on firmer ground with nonparametric likelihood" (Section 7).

Let us examine these assertions more closely. Statement (a), that the bootstrap is not automatic, is surely true, more deeply than Young discusses. Data does not follow a statistical model. Random variables are a mathematical construct, as are stationary time series and more complex models. The goal of statistical theory is to analyze procedures in hypothetical situations that mimic aspects of data. Even the most complete theory is easily misapplied. The first part of statement (f) founders on this reality. The use of bootstrap or other statistical procedures, like the use of surgical instruments, is an *empirical* business that offers no guarantees or refunds. This does not preclude success in skilled hands.

Statement (b) hastens to judge a very active topic. The modification of bootstrap resampling size has received closer scrutiny in recent technical reports by D. Politis and J. Romano and in a prominent invited lecture by F. Götze at the 1993 Annual Meeting of the IMS. The study of the wild bootstrap and generalized bootstrap is likewise moving ahead rapidly, for instance, in work by E. Mammen. Each of these strategies handles examples where simple bootstrapping fails. Early numerical results support the theory.

Statement (c) can be set against the prehistory of

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