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ASYMPTOTICALLY NORMAL FAMILIES OF DISTRIBUTIONS AND EFFICIENT ESTIMATION

BY I. A. IBRAGIMOV AND R. Z. KHAS'MINSKII

Leningrad Branch of the Steklov Mathematical Institute and Institute of Information Transmission

1. Introduction. In parametric estimation theory a very important role is played by the notion of local asymptotic normality (LAN), an idea introduced by Le Cam [Le Cam (1953, 1956, 1960)]. In particular, this theory establishes very general lower bounds on the accuracy of estimates [Le Cam (1953, 1972) and Hájek (1972, 1970), theorems]. Below in all references to the LAN theory, we follow our treatment of the theory [Ibragimov and Has'minskii (1981)]. A different treatment can be found in Le Cam (1986), Chapters 7 and 8.

It seems that Levit was the first to understand the importance of the LAN concept for nonparametric estimation theory [see Levit (1974, 1975b)]. He also showed that the corresponding lower bounds can be attained in some infinite-dimensional estimation problems [Levit (1978)]. Further advances and generalizations of these results were obtained by Millar (1983). The first part of this chapter looks at the investigations of Levit and Millar from a new point of view and may be considered as an infinite-dimensional variant of Ibragimov and Has'minskii (1981), Chapter 2. We suggest a new [different from Levit (1978) or Millar (1983)] definition of LAN for families of distributions $\{P_\theta^{(\varepsilon)}, \theta \in \Theta\}$, where the parametric set Θ is a subset of a normed space (Section 2) or a smooth infinite-dimensional manifold (Section 5).

For families $\{P_\theta^{(\varepsilon)}, \theta \in \Theta\}$ satisfying the LAN condition with an infinite-dimensional parametric set Θ, we consider the following estimation problem. We would like to estimate the value $\phi(\theta)$ of a known (Euclid- or) Hilbert-valued function $\phi(\cdot)$ at an unknown point $\theta \in \Theta$ on the basis of observations $X^{(\varepsilon)}$ corresponding to the family $\{P_\theta^{(\varepsilon)}, \theta \in \Theta\}$. Although this is a rather nonparametric estimation problem, we may also consider it as a problem of specifying a plausible value for the parameter ϕ in the presence of an infinite-dimensional nuisance parameter θ. Instead, we may treat the problem as a semiparametric estimation problem [see Begun, Hall, Huang and Wellner (1983) and Wellner (1985)]. We prove under our LAN conditions a variant of Hájek's convolution theorem (Sections 3 and 5) and a variant of the Hájek–Le Cam asymptotic minimax bound. The latter result enables us to define the

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