This leads to an estimate of $f(y|\mathbf{x})$ having the form

$$(1) \qquad \hat{f}(y|\mathbf{x}) = \exp\left(\sum_k \sum_j \hat{\beta}_{jk} H_{jk}(\mathbf{x}) B_k(y) - c\big(\mathbf{h}(\mathbf{x};\hat{\boldsymbol{\beta}})\big)\right), \qquad y \in \mathscr{Y},$$

where $\hat{\beta}$ is the JK-tuple consisting of $\hat{\beta}_{jk}$, $1 \le k \le K$ and $1 \le j \le J_k$, in some order. This estimate has the form of a multiparameter exponential family, so the corresponding log-likelihood function is again concave. The asymptotic theory of such estimates, with $\mathscr{Y}$ a compact interval in $\mathbb{R}$, $\mathscr{H}_1 = \cdots = \mathscr{H}_K$ and bases consisting of B-splines and without model selection, has been treated in Stone (1989). It remains to investigate the numerical behavior of such estimates, especially as modified to incorporate the strategy of MARS. Perhaps the resulting technology should be referred to as multivariate adaptive response splines (MARES).

Suppose, in particular, that $\mathscr{Y} = \{0, 1\}$. Then we can let $\mathscr{S}$ be the one-dimensional space having basis $B_1(y) = y$. In this context, (1) reduces to logistic regression. Similarly, by letting $\mathscr{Y}$ be a finite set of size 3 or more, we can apply the strategy of MARS to the polytomous extension of logistic regression.

The more general setup given by (1) allows for the estimation of the conditional variance and conditional quantiles of an arbitrary random variable Y given $\mathbf{X}$ as well as estimation of the conditional mean of Y given $\mathbf{X}$, which is treated in the present paper.

The general strategy of MARS is also applicable to time series.

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