BALANCED REPEATED MEASUREMENTS DESIGNS

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It was pointed out by Joachim Kunert that there are mistakes in the proofs of Theorem 4.3 and Theorem 4.4. They are now corrected as follows.

(I) *Theorem* 4.3. The C_{ij}-matrices in the original proof (5.7) are for uniform designs. For designs that are uniform on units and the last period,

$$C_{d11} = \lambda_1 p I_t - n^{-1}[\sum_{k=1}^{p} \ell_{ik}\ell_{jk}]_{i,j},$$

$$C_{d12} = M_d - n^{-1}[\sum_{k=2}^{p} \ell_{ik}\ell_{j,k-1}]_{i,j},$$

and

$$C_{d22} = \lambda_1(p - 1 - p^{-1})I_t + \lambda_1 p^{-1}t^{-1}J_t - n^{-1}[\sum_{k=1}^{p-1} \ell_{ik}\ell_{jk}]_{i,j}.$$

However the original conclusion of Theorem 4.3 is still valid. It can be shown that for any d

$$C_{d22} \le C_{d^*22} = \lambda_1(p - 1 - p^{-1})(I_t - t^{-1}J_t).$$

Therefore the maximization of tr C_d = tr C_{d11} − tr $C_{d12}C_{d22}^{-}C_{d21}$ can be replaced by the maximization of

$$\text{tr } C_{d11} - \text{tr } C_{d12}C_{d^*22}^{-}C_{d21} = \lambda_1 pt - n^{-1}\sum_{i=1}^{t}\sum_{k=1}^{p}\ell_{ik}^2 - \lambda_1^{-1}(p - 1 - p^{-1})^{-1}\sum_{i,j=1}^{t}(m_{ij} - n^{-1}\sum_{k=2}^{p}\ell_{ik}\ell_{j,k-1})^2$$

subject to the same constraints in the proof of Theorem 4.1. The above expression is somewhat simpler than (5.5) in the proof of Theorem 4.1.

That the expression is maximized by the uniform design d^* can be proved by almost the same arguments as employed in the proof of Theorem 4.1. The details are omitted.

(II) *Theorem* 4.4. There is a technical error in the proof of the first part of the Theorem. The row sums of the $\tilde{C}$-matrix are in general not equal to zero, thus making Kiefer's Theorem inapplicable. In fact for designs that are equi-replicated and are uniform on the first and last periods, the row sums are equal to the positive constant $\lambda_1(p - 1)/p$. A simple way of repairing Theorem 4.4 was suggested by Dr. Kunert. Reparametrize the model (1.1) (without the period effect) by imposing an additional constraint

$$\sum_{i=1}^{t}\rho_i = 0.$$

It is easy to verify that the new $\tilde{C}$-matrix has zero row sums. The rest of the proof can be obtained by imitating the proof of Theorem 4.1. See also the proof of Theorem 4.4.

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