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CORRECTION TO "THE EXISTENCE AND UNIQUENESS OF STATIONARY MEASURES FOR MARKOV-RENEWAL PROCESSES"

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There is a defect in the proof of the uniqueness of π given in [1]. On page 1452, lines 24 and 25, it is stated that $M_{jj}^*(t) = M_{jj}(t)$ implies $G_{jj}^*(t) = G_{jj}(t)$. The former does imply that each state in the time-reversed process is recurrent but says nothing about the communication of the various states in the time-reversed process. Thus, there is no reason for $G_{ij}^*(+\infty) = 1$ for $i \neq j$ and, in fact, $G_{ij}^*(t)$ has not been defined. We rectify that here.

The proper definition of $G_{ij}^*(t)$ is (see [2])

$$\begin{aligned} G_{ij}^*(t) &= c_i^{-1} c_j H_i * \textstyle\sum_{k\neq j} p_{jk\ j}M_{ki}(t) \qquad &\text{if} \quad i \neq j \\ &= G_{jj}(t) &\text{if} \quad i = j. \end{aligned}$$

Thus, $G_{ij}^*(+\infty) = c_i^{-1} c_j \sum_{k\neq j} p_{jk\ j}M_{ki}(+\infty) = c_i^{-1} c_{j\ j}M_{ji}(+\infty) > 0$ and all states communicate in the time-reversed process so that it, too, is irreducible.

Without loss of generality, we set $c_0 = 1$. Then, letting $j = 0$, we have $1 \geq G_{i0}^*(+\infty) = c_i^{-1}\ {}_0M_{0i}(+\infty) = c_i^{-1} m_i$. Thus, $c_i \geq m_i$ for all $i \in I^+$ and $c_i - m_i$ is a nonnegative solution of

$$x_j = \textstyle\sum_{i,k} x_i p_{ik}(1 - H_i) * M_{kj}(t)$$

for all $t > 0$ that vanishes at $i = 0$. Such a solution must vanish identically and $c_i = m_i$ for $i \in I^+$.

Two more proofs of the uniqueness of π are given in [2]. One involves using the π time-reversed process. The other does not use time-reversals and hence can be adapted to processes that are more general, e.g., those that may not have a last state or even a next state.

REFERENCES

[1] PYKE, RONALD and SCHAUFELE, RONALD (1966). The existence and uniqueness of stationary measures for Markov renewal processes. *Ann. Math. Statist.* **37** 1439–1462.

[2] SCHAUFELE, RONALD A. (1980). A class of time-reversible semi-Markov processes. Unpublished manuscript.

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