

ON THE TCHEBYCHEF INEQUALITY OF BERNSTEIN

By

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From Tchebychef's inequality we know that if x_1, x_2, \dots, x_n are a set of independent statistical variables with

$$m_{x_1} = m_{x_2} = \dots = m_{x_n} = 0,$$

and

$$\sigma^2 = \sigma_{x_1}^2 + \sigma_{x_2}^2 + \dots + \sigma_{x_n}^2,$$

then the probability P that

$$-t\sigma \leq x_1 + x_2 + \dots + x_n \leq t\sigma$$

satisfies the inequality,

$$P \geq 1 - \frac{1}{t^2}.$$

This gives a lower limit for P which is often unsatisfactory. Improvement of this result requires further hypotheses. As is well-known, Pearson, Camp, Guldberg, Meidel, Narumi,² and Smith³ have attacked this problem with considerable success. Another interesting and important attempt in this direction due to S. Bernstein seems to have generally escaped attention in the English-speaking world, at least, since it has been published only in Russian.⁴ Because of the latter fact, it seems necessary to give

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²For references to all these papers except Smith's and a brief discussion see Rietz, H. L., *Mathematical Statistics*, (Open Court Publishing Company, Chicago, 1927), pp. 140-144.

³Smith, C. D., *On Generalized Tchebychef Inequalities in Mathematical Statistics*, *American Journal of Mathematics*, Vol. 52, (1930), pp. 109-126.

⁴Bernstein, S., *Theory of Probability*, (Moscow, 1927), pp. 159-165. The present account of this work of Bernstein is taken from a lecture of Professor J. V. Uspensky.