NOTES

ON STANDARD ERROR FOR THE LINE OF MUTUAL REGRESSION

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1. In Pearson's *On Lines and Planes of Closest Fit to System of Points in Space*, he establishes a formula for the mean square residual for the best fitting line in q-space:

$$(1) \qquad \text{(mean sq. residual)}^2 = \sigma_{x_1}^2 + \cdots + \sigma_{x_q}^2 - \Delta R_{\max}^2$$

where $2R_{\max}$ is the length of the maximum axis of the correlation ellipse in q-space, and Δ is the correlation determinant.[1]

In the present paper, we consider a 2-dimensional case, and shall call the mean sq. residual as the standard error, denoted by S_N.

In 2-dimensional space, a correlation ellipse is

$$(2) \qquad ax^2 + 2hxy + by^2 + c = 0,$$

where

$$(2a) \quad a = \sigma_y^2, \quad b = \sigma_x^2, \quad h = -r_{xy}\sigma_x\sigma_y = -p_{xy} = -p_{yx}, \quad c = -\sigma_x^2\sigma_y^2.$$

Pearson gives in the 2-dimensional space the following formula for S_N:

$$(3) \qquad S_N = \sigma_x\sigma_y/\text{semi-major axis of equation (2)}.$$

Expression (3) can be readily deduced from (1). This paper aims to present some formulae for S_N, more convenient for practical computation, and also call attention to a misprint in Pearson's paper.

2. From analytic geometry, we see that the angle φ, between the major axis of the ellipse (2) and the x-axis is given by

$$(4) \qquad \tan 2\varphi = 2h/(a - b).$$

By rotation of the axes, equation (1) can be written in the form

$$(5) \qquad a'x^2 + b'y^2 + c = 0,$$

where

$$(5a) \qquad \begin{aligned} a' &= a\cdot\cos^2\varphi - 2h\cdot\sin\varphi\cdot\cos\varphi - b\cdot\sin^2\varphi > 0 \\ b' &= a\cdot\sin^2\varphi - 2h\cdot\sin\varphi\cdot\cos\varphi - b\cdot\cos^2\varphi > 0. \end{aligned}$$

[1] Philosophical Magazine, 6th Series, II (November, 1901), p. 559.