Let

$$f_j^*(v) = f_j(v)e^{-\frac{1}{2}\Sigma v_i^2} \qquad (j = 1, 2). \tag{19}$$

Now we shall show that for any positive values $\beta_1, \cdots, \beta_k$

$$\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} f_j^*(v_1, \cdots, v_k)e^{\beta_1|v_1|+\cdots+\beta_k|v_k|}\, dv_1 \cdots dv_k < \infty. \tag{20}$$

In fact, consider the 2^k sets $(a_1, \cdots, a_k)$ where $a_i = \pm 1$ $(i = 1, \cdots, k)$. Denote by $R_{a_1 \cdots a_k}$ the subset of the k-dimensional Cartesian space which consists of all points $v = (v_1, \cdots, v_k)$ for which v_i is either zero or signum $v_i =$ signum a_i $(i = 1, \cdots, k)$. Putting $\alpha_i = a_i\beta_i$, it follows from (17) and (18) that

$$\int_{R_{a_1 \cdots a_k}} f_j^*(v_1, \cdots, v_k)e^{\beta_1|v_1|+\cdots+\beta_k|v_k|}\, dv_1 \cdots dv_k < \infty. \tag{21}$$

Since (21) holds for any of the 2^k sets $R_{a_1 \cdots a_k}$, equation (20) is proved.

From (1) it follows that

$$\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} v_1^{r_1} \cdots v_k^{r_k} [f_1^*(v_1, \cdots, v_k) - f_2^*(v_1, \cdots, v_k)]\, dv_1 \cdots dv_k = 0,$$

for all non-negative integers $r_1, \cdots, r_k$. Hence, because of (21) and Lemma A we see that

$$f_1^*(v_1, \cdots, v_k) = f_2^*(v_1, \cdots, v_k), \tag{22}$$

except perhaps on a set of measure zero. From (22) it follows that

$$f(v_1, \cdots, v_k) = f_1(v_1, \cdots, v_k) - f_2(v_1, \cdots, v_k) = 0,$$

except perhaps on a set of measure zero. Hence Proposition I is proved.

A NOTE ON SKEWNESS AND KURTOSIS

By J. Ernest Wilkins, Jr.

University of Chicago

It is the purpose of §1 of this paper to prove the following inequality:

$$\alpha_4 \geqq \alpha_3^2 + 1. \tag{1}$$

This inequality seems to have first been stated by Pearson[1]. The inequality also follows from a result appearing in the thesis of Vatnsdal. Here we give a proof based on the theory of quadratic forms which seems to be more direct and more elementary than either of the previous proofs.

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[1] "Mathematical contributions to the theory of evolution, XIX; second supplement to a memoir on skew variation," *Phil. Trans. Roy. Soc.* (*A*), Vol. 216 (1916), p. 432.