The Annals of Mathematical Statistics
1968, Vol. 39, No. 5, 1770–1795

ABSTRACTS OF PAPERS

(*Abstracts of papers presented at the Annual meeting, Madison, Wisconsin, August 26–30, 1968. Additional abstracts appeared in earlier issues.*)

25. Optimal and efficient designs of experiments. Corwin L. Atwood, University of Minnesota.

We estimate s out of k regression coefficients. If $s = k$ and the regression functions are of low degree in each variable on a compact finite dimensional space satisfying certain symmetries, then any D-optimal design is supported only on certain points of symmetry. If $s < k$ we give a sufficient condition for D-optimality in terms of convergent sequences of designs, and we give a condition equivalent to D-optimality [modifying Karlin and Studden, *Ann. Math. Statist.* **37** 800–807]. Application of the above to multilinear regression on the simplex characterizes, and in some cases determines, optimal designs. An upper bound is known for the number of points of support necessary for a D-optimal design, $s \leqq k$; we show that the bound is sharp. For $s \leqq k$, we give a lower bound on the D-efficiency of the best design on k or fewer points. If $s = k$ we give a sharp lower bound on the G-efficiency of the best k-point design. Inequalities in both directions are given relating the D- and G-efficiencies of any design, for $s \leqq k$. The inequality in one direction is sharp. Sharp lower bounds are given for the D- and G-efficiencies of a single design used in several models. (Received 24 June 1968.)

6. A controlled transportation queueing process. U. Narayan Bhat, Case Western Reserve University.

A transportation queueing process in which taxis arrive in a Poisson process and customers arrive as a renewal process independent of the taxi-arrival process is controlled by calling extra taxis whenever the total number of customers lost to the system reaches a certain pre-determined number. Transient and steady state behavior of this process is studied using renewal theoretic arguments. A method is proposed to determine the best value of the control variable so as to balance the cost to a taxi due to waiting against the cost to a customer. Results given here are extensions of those obtained by Bhat and Erickon [(1966), RM 160, Department of Statistics, Michigan State University] in an inventory system context. (Received 3 July 1968.)

27. Asymptotically optimal sequences in exponential subhouses. M. C. Bhattacharjee, University of California, Berkeley.

Let A be a given Borel subset of the set of all probabilities on the real line. Any sequence $\{Y_n : n = 1, 2, \cdots\}$ is called an A-sequence if $\mathfrak{L}(Y_1) \varepsilon A$, $\mathfrak{L}(Y_{n+1} \mid Y_1, \cdots, Y_n) \varepsilon A$ for all n. For $a > 0$, the set of probabilities $\Gamma(a) = \{\gamma : \int \exp(ay)\gamma(dy) \leqq 1\}$ is called an exponential house. For A-sequences, the optimal probability that some partial sum exceeds $s > 0$ is $P_A(s) = \sup P(Y_1 + \cdots + Y_n \geqq s, \text{ some } n)$, the supremum being over all A-sequences. It is well known that $P_{\Gamma(a)}(s) = \exp(-as)$. If the set A of available distributions is smaller, this upper bound for $P_A(s)$ may not be attained. A is an exponential subhouse if A is contained in some $\Gamma(a)$. In this paper we find sufficient conditions under which the smallest such exponential bound is asymptotically attained, using dynamic programming methods. One of the sufficient conditions may be stated as follows: Let $\Gamma^0(a) = \{\gamma : \int \exp(ay)\gamma(dy) = 1\}$. For an exponential subhouse A, let $\Gamma(\alpha_A)$ be the smallest ex-