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DISCUSSION OF "FIBER DIRECTION ESTIMATION IN DIFFUSION MRI"

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1. Introduction. The paper by Wong, Lee, Paul and Peng offers a telling example of experimentalist vs. statistical thinking. While the experimentalists in the imaging world proposed to solve the problem of identifiability of the multi-tensor model by making more measurements at higher spatial resolution and at multiple magnetic gradient strengths, Wong et al. asked if estimation was still possible in the usual experimental setting and offered a different solution. By choosing the right parametrization, they have shown that, while the full multi-tensor model cannot be estimated, the diffusion directions can. If the goal of the analysis is tractography, then this is sufficient.

The paper is comprehensive, taking the analysis all the way from the measured diffusion directions to fiber tracking. Estimation of the diffusion directions is followed by spatial smoothing to improve accuracy. Clustering is used so that smoothing is only applied within fibers where the angle between neighboring voxels is small. This helps the fiber tracking algorithm better survive the difficult fiber crossing regions.

Because the objects of the analysis are diffusion directions, it is of interest to connect the work by Wong et al. to the existing body of knowledge of directional statistics. In the rest of this comment, I explore how directional statistics may shed some additional light on the problem.

2. The multi-tensor model. A key idea in the paper is that, while the full multi-tensor model is unidentifiable, the tensor eigenvectors are not. The identifiability problem is hard to spot at first because the multi-tensor model [equation (1) in Wong et al.] looks very much like a mixture of Gaussians. Gaussian mixture models are generally identifiable unless there is degeneracy in the parameters. The model is deceiving, however, because the argument vector $\mathbf{u}$ is constrained to have unit norm. In fact, if the displacement of water molecules is modeled as a Gaussian mixture distribution, then the signal model in equation (1) is proportional to the Fourier transform of this distribution, restricted to a sphere of constant radius in frequency space [Mori (2007)]. The manipulation by Scherrer and Warfield (2010) (described by Wong et al. in Section 3.1) indicates that it is possible to modify the diffusion tensors in the original Gaussian mixture together with the mixture

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