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ON SOME PROBLEMS IN THE ARTICLE *EFFICIENT LIKELIHOOD ESTIMATION IN STATE SPACE MODELS* BY CHENG-DER FUH [*Ann. Statist.* 34 (2006) 2026–2068]

BY JENS LEDET JENSEN

University of Aarhus

1. Introduction. Upon reading the paper *Efficient Likelihood Estimation in State Space Models* by Cheng-Der Fuh I found a number of problems in the formulations and a number of mathematical errors. Together, these findings cast doubt on the validity of the main results in their present formulation. A reformulation and new proofs seem quite involved.

The paper, *Efficient Likelihood Estimation in State Space Models* deals with asymptotic properties of the maximum likelihood estimate in hidden Markov models. The hidden Markov chain is X_n, and the observed process is ξ_n where ξ_n conditioned on the past and the hidden process depends on (X_n, ξ_{n-1}) only. The approach used is to add an iterated function system M_n, and to consider the Markov process (X_n, ξ_n, M_n). This is very much akin to the method in Douc and Matias [1], and I will use this article as a background for my comments.

2. Problems.

2.1. *Definition of iterated function system.* The first basic definition in the paper is a function $\mathbf{P}_\theta(\xi_j) : \mathbf{M} \to \mathbf{M}$ that maps a function $h \in \mathbf{M}$ into a new function in $\mathbf{M}$ (page 2031),

$$\mathbf{P}_\theta(\xi_j)h(x) = \int_{y\in\mathcal{X}} p_\theta(x, y) f(\xi_j; \theta|y, \xi_{j-1})h(y)m(dy).$$

[It is unclear why the author states that $\mathbf{P}_\theta(\xi_j)$ is a function on $(\mathcal{X} \times \mathbf{R}^d) \times \mathbf{M}$ where $\mathcal{X}$ is the state space of the Markov chain.] The paper next defines the composition $\mathbf{P}_\theta(\xi_{j+1}) \circ \mathbf{P}_\theta(\xi_j)h$ by first applying $\mathbf{P}_\theta(\xi_{j+1})$ to h and then applying $\mathbf{P}_\theta(\xi_j)$ to the result. Using these two definitions we have

$$\mathbf{P}_\theta(\xi_n) \circ \cdots \circ \mathbf{P}_\theta(\xi_1) \circ \mathbf{P}_\theta(\xi_0)\pi_\theta$$
$$= \int \pi_\theta(x_n)\left\{\prod_{j=n}^{1} p_\theta(x_{j-1}, x_j) f(\xi_j; \theta|x_j, \xi_{j-1})m(dx_j)\right\} f(\xi_0; \theta|x_0)m(dx_0).$$

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