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REJOINDER

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We heartily thank all discussants for their thoughtful and friendly comments on our paper. As the three discussions address distinct issues, with very little overlap, this rejoinder is also organized into three distinct parts.

1. Discussion by Serfling and Zuo.

1.1. *Point-valued versus hyperplane-valued quantiles*. Univariate quantiles are point-valued. Points on the real line $\mathbb{R}^1$ are hyperplanes of dimension 0, and hence, in a k-dimensional context, defining quantiles as points or as $(k-1)$-dimensional hyperplanes may be equally legitimate and, depending on the properties to be emphasized or the inference problem at hand, equally sensible and useful.

If L-estimation is to be privileged, point-valued quantiles, which provide easily tractable integrands for L-functionals, may look more attractive. Many basic properties of quantiles, however, are better accounted for by hyperplane-valued generalizations. In particular, the role of univariate quantiles as critical values partitioning, with respect to outlyingness, the sample or the observation space into two regions with given frequencies or probability contents cannot be assumed, in $\mathbb{R}^k$, by any point-valued concept. Since our objective was to establish a meaningful and fully operational bridge between the quantile and depth universes, hyperplane-valued concepts quite naturally came into the picture; this does not imply, however, that point-valued quantiles should be abandoned in favor of hyperplane-valued ones.

1.2. *Robustness issues*: *A useful adjunct to the halfspace DOQR paradigm*. The discussants point out that our quantile hyperplane construction, since it leads to halfspace depth, also inherits all the properties of the halfspace depth DOQR (Depth, Outlyingness, Quantile, Rank) paradigm. Among them, they are stressing [point (H2) of their discussion] a poor performance in outlier detection.

Since no robustness requirement has been made at any stage in our construction of directional quantiles, such poor performance is not surprising. Our quantile hyperplanes, however, do not come *alone*: thanks to their L_1 nature, and their relation to linear programming, each $\pi_{\tau\mathbf{u}}^{(n)}$ indeed is accompanied by a Lagrange multiplier

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