

DISCUSSION

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1. Introduction. Hallin, Paindaveine and Šiman—hereafter HPŠ—are to be congratulated for the appreciation their paper is receiving from *The Annals of Statistics*. We personally are indebted for the attention they devoted to our paper, [11]—hereafter KM. The topic is quite delicate and multifaceted, hence all the misunderstandings we allege below show deficiencies of our exposition rather than anything else.

2. This is not a quantile. The altered title of Magritte’s painting entitling this section can be seen as a gnomic expression of some subtle, nevertheless fundamental differences between HPŠ and KM, a distinct “philosophy” in the language of HPŠ. While HPŠ state in their first sentence their intent to “propose a definition of multivariate quantiles,” KM in their second sentence maintain

While such an objective could be mistaken for yet another attempt in the ongoing quest for “multivariate quantiles”... we would like to stress that we differ in the position that *no multivariate generalization of the quantile concept may be needed at all*—...

Instead, our intention was to discuss “certain aspects of using quantiles to obtain insights about multivariate data,” suggesting that interesting insights about multivariate data can be obtained by looking at *univariate* quantiles of projections. Nothing beyond concepts already well established in data-analytic practice: see [12] regarding univariate quantiles (the concept we feel primarily rooted in the order of the numerical scale rather than in their secondary L^1 characterization), and any textbook on multivariate analysis regarding projections.

We admit that KM might have not accentuated the word “univariate” enough—nevertheless, nothing like “projection quantiles” is mentioned, nor anything like τ -quantile or $\mathbf{q}_{\text{KM};\nu\mathbf{u}}$ is introduced there. Put verbally, to escape sometimes serpentine notation: KM *do not consider lines orthogonal to the quantile directions* “*multivariate quantiles*,” but merely consider them a graphical tool to visualize *univariate* quantiles of the corresponding *projections*. If HPŠ write

“The resulting quantile contours (the collections, for fixed τ , of $\mathbf{q}_{\text{KM};\nu\mathbf{u}}$ ’s do not enjoy the properties (independence with respect to the choice of an origin, affine-equivariance, nestedness, etc.) one is expecting from a quantile concept.”

Received August 2009.

¹Supported by the Natural Sciences and Engineering Research Council of Canada.