

DISCUSSION

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With delight we most heartily congratulate Hallin, Paindaveine and Šiman (HPS) on a superb and stimulating paper. It uniquely impacts our thinking about regression quantiles, multivariate quantiles, and the halfspace depth. Here we examine this highly significant contribution from the standpoints of some perspectives on multivariate quantile and depth functions, some criteria to consider in choosing such functions, and some further points about the much-studied halfspace depth. We also raise a few technical issues and questions for consideration.

General perspectives on quantile and depth functions. In thinking about any new contribution to multivariate quantile functions, we may draw upon the following perspectives, which also clarify the univariate case in some respects.

- (P1) *In multivariate analysis, orientation to a “center” compensates for lack of a natural order.*
- (P2) *In the context of quantiles, the role of “center” is naturally given to the “median.”*
- (P3) *The inverse of a quantile function is not the distribution F but rather the rank function.*
- (P4) *Depth, outlyingness, quantile, and rank functions are equivalent (DOQR paradigm).*
- (P5) *Quantile functions are best viewed as parameters or characteristics of the distribution F .*
- (P6) *Equivalence between distribution and quantile functions is not an essential requirement.*

Let us briefly elaborate on some of these points.

(P3). In the univariate case, a natural linear order makes it convenient and straightforward to define distribution and quantile functions as mutual inverses, F and F^{-1} . However, for extension to higher dimension, the equivalent *median-oriented* formulation is the most appropriate point of departure. That is, via $u = 2p - 1$, the usual quantile function $F^{-1}(p)$, $0 < p < 1$, may be represented

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