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Comment: Expert Elicitation for Reliable System Design

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INTRODUCTION

The paper by Bedford, Quigley and Walls has reviewed the role of expert judgements in reliability assessment within the engineering design process. It differs from many statistically based papers that focus mainly on statistical models, in that it addresses issues related to the process, the concept and principles. The paper itself is comprehensive in coverage and offers general guidance for reliability assessment at the system design stage. However, to some extent, the technical side of the statistical techniques used in expert opinion elicitation in reliability analysis has not been fully explored. A practitioner who wants to conduct research in the area still needs to search methodologies from other recommended sources in the paper to find practical solutions. This is perhaps the purpose of the paper.

In my discussion, two techniques that are particularly useful to subjective data elicitation, but are not discussed in the Bedford, Quigley and Walls paper, namely, empirical Bayes (EB; Robbins, 1955) and evidential reasoning (ER; Dempster, 1966), will be briefly discussed. It is hoped that this will shed some light on the problem of expert data elicitation in general and reliability assessment in design in particular.

THE EB AND ER APPROACHES

Suppose that a prior distribution is hypothesized for the parameter Θ, but is only specified to be in a certain class of prior distributions. We can represent this by a prior distribution $p(\Theta|\Phi)$, where the hyperparameters Φ index the family of priors. We can then construct a "likelihood"

$$p(x|\Phi) = \int p(x|\Theta)p(\Theta|\Phi)\,d\Theta \tag{1}$$

that relates the data to the hyperparameters. In the so-called empirical Bayes approach, Φ is estimated from (1) by classical methods such as unbiased estimation, yielding an estimate $\hat{\Phi}$. The prior is then taken to be $p(\Theta|\hat{\Phi})$ and the inference about Θ is by the appropriate Bayes rule for this prior. The analysis thus has the flavor of a Bayesian analysis, but with an empirical prior based on the data, so it is termed empirical Bayes. If we view x as the subjective or expert data, then the EB approach can be readily used in the reliability analysis of design problems.

The Dempster–Shafer theory of evidence (Dempster, 1966; Shafer, 1976) was particularly developed to aggregate subjective assessments in decision-making. To begin with, suppose there is a simple two-level hierarchy of attributes with a general attribute at the top and a number of basic attributes at the bottom. In the reliability assessment of a designed system, the basic attributes can be viewed as the components of the system and the general attribute can be the system reliability. Let $P_{n,i}$ be a probability mass that represents the degree to which the ith basic attribute supports the hypothesis that the general attribute is assessed to the nth grade. Then $P_{n,i}$ is calculated as

$$P_{n,i} = \omega_i \beta_{n,i}, \tag{2}$$

where ω_i is the weight of the ith attribute and $\beta_{n,i}$ is the degree of belief that the ith basic attribute supports the hypothesis that the general attribute is assessed to the nth grade. Define a subset of the basic attributes and let $P_{n,I(i)}$ denote the probability mass defined as the degree to which all the attributes in the subset support the hypothesis that the general attribute is assessed to the nth grade. Then we have

$$P_{n,I(i+1)} = K_{I(i+1)} P_{n,I(i)} P_{n,i+1}, \tag{3}$$

where $K_{I(i+1)}$ is a normalizing factor. In the original ER approach, the combined degree of belief β_n is given by

$$\beta_n = P_{n,I(L)}, \tag{4}$$

where L is the number of basic attributes.

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