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# Rejoinder: A Selective Overview of Nonparametric Methods in Financial Econometrics

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I am very grateful to the Executive Editor, Edward George, for organizing this stimulating discussion. I would like to take this opportunity to thank Professors Peter Phillips, Jun Yu, Michael Sørensen, Per Mykland and Lan Zhang for their insightful and stimulating comments, touching both practical, methodological and theoretical aspects of financial econometrics and their applications in asset pricing, portfolio allocation and risk management. They have made valuable contributions to the understanding of various financial econometric problems.

The last two decades have witnessed an explosion of developments of data-analytic techniques in statistical modeling and analysis of complex systems. At the same time, statistical techniques have been widely employed to confront various complex problems arising from financial and economic activities. While the discipline has grown rapidly over the last two decades and has rich and challenging statistical problems, the number of statisticians involved in studying financial econometric problems is still limited. In comparison with statisticians working on problems in biological sciences and medicine, the group working on financial and econometric problems is dismally small. It is my hope that this article will provide statisticians with quick access to some important and interesting problems in financial econometrics and to catalyze the romance between statistics and finance. A similar effort was made by Cai and Hong [12], where various aspects of nonparametric methods in continuous-time finance are reviewed. It is my intention to connect financial econometric problems as closely to statistical problems as possible so that familiar statistical tools can be employed. With this in mind, I sometimes oversimplify the problems and techniques so that key features can be highlighted.

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I am fully aware that financial econometrics has grown into a vast discipline itself and that it is impossible for me to provide an overview within a reasonable length. Therefore, I greatly appreciate what all discussants have done to expand the scope of discussion and provide additional references. They have also posed open statistical problems for handling nonstationary and/or non-Markovian data with or without market noise. In addition, statistical issues on various versions of capital asset pricing models and their related stochastic discount models [15, 19], the efficient market hypothesis [44] and risk management [17, 45] have barely been discussed. These reflect the vibrant intersection of the interfaces between statistics and finance. I will make some further efforts in outlining econometric problems where statistics plays an important role after brief response to the issues raised by the discussants.

## 1. BIASES IN STATISTICAL ESTIMATION

The contributions by Professors Phillips, Yu and Sørensen address the bias issues on the estimation of parameters in diffusion processes. Professors Phillips and Yu further translate the bias of diffusion parameter estimation into those of pricing errors of bonds and bond derivatives. Their results are very illuminating and illustrate the importance of estimation bias in financial asset pricing. Their results can be understood as follows. Suppose that the price of a financial asset depends on certain parameters $\theta$ (the speed of the reversion $\kappa$ in their illustrative example). Let us denote it by $p(\theta)$, which can be in one case the price of a bond and in another case the prices of derivatives of a bond. The value of the asset is now estimated by $p(\hat{\theta})$ with $\hat{\theta}$ being estimated from empirical data. When $\hat{\theta}$ is overestimated (say), which shifts the whole distribution of $\hat{\theta}$ to the left, the distribution of $p(\hat{\theta})$ will also be shifted, depending on the sensitivity of $p$ to $\theta$. The sensitivity is much larger for bond derivatives when $\kappa$ is close to zero (see Figure 2 of [46]), and hence the pricing errors are much larger. On the other hand, as the distribution