Statistical Science
2005, Vol. 20, No. 4, 367–371
DOI 10.1214/088342305000000403

Comment: Randomized Confidence Intervals and the Mid-P Approach

Alan Agresti and Anna Gottard

We enjoyed reading the interesting, thought-provoking article by Geyer and Meeden. In our comments we will try to place their work in perspective relative to the original proposals for exact and randomized confidence intervals for the binomial parameter. We propose a fuzzy version of the original binomial randomized confidence interval, due to Stevens (1950). Our approach motivates an existing nonrandomized confidence interval based on inverting a test using the mid-P value. The mid-P confidence interval provides a sensible compromise that mitigates the effects of conservatism of exact methods, yet provides results that are more easily understandable to the scientist.

1. HISTORICAL PERSPECTIVE

Clopper and Pearson (1934) proposed the following $100(1-\alpha)$% confidence interval for a binomial parameter θ: The bounds $[\theta_L, \theta_u]$ are the solutions to the equations

$$\sum_{k=0}^{x} \binom{n}{k} \theta_U^k (1-\theta_U)^{n-k} = \alpha/2$$

and

$$\sum_{k=x}^{n} \binom{n}{k} \theta_L^k (1-\theta_L)^{n-k} = \alpha/2.$$

(One takes $\theta_L = 0$ when $x = 0$ and $\theta_U = 1$ when $x = n$.) This confidence interval is based on inverting two one-sided binomial tests. Because of discreteness, the method is conservative; the actual confidence level is bounded below by $1-\alpha$ (Neyman, 1935).

To eliminate the conservativeness, Stevens (1950) suggested instead solving the binomial-probability equations

$$\Pr_{\theta_U}(X < x) + U \times \Pr_{\theta_U}(X = x) = \alpha/2$$

and

$$\Pr_{\theta_L}(X > x) + (1-U) \times \Pr_{\theta_L}(X = x) = \alpha/2,$$

where U is a Uniform$(0, 1)$ random variable. This confidence interval is based on inverting tests for which (as in the case of continuous random variables) the one-sided P-values have a uniform null distribution and sum to 1, unlike the ordinary one-sided P-values used in the Clopper–Pearson confidence interval. We will refer to this as the *Stevens randomized confidence interval*. Anscombe (1948) made the analogous one-sided proposal of inverting a randomized one-sided binomial test so as to obtain an upper or lower randomized confidence bound. Blyth and Hutchinson (1960) provided tables for implementing a slightly different randomized confidence interval (proposed by M. W. Eudey in a 1949 technical report at the University of California, Berkeley) that has the property of being Neyman shortest unbiased.

These days statisticians regard randomized inference as a tool for the mathematical convenience of achieving exactly the desired size or confidence level with discrete data, but they do not consider actually implementing it in practice. However, this method was originally thought to have considerable promise.

For example, Pearson (1950) suggested that statisticians may come to accept randomization after performing an experiment just as they had gradually come to accept randomization for the experiment itself. He predicted that randomized confidence intervals "will not meet with strong objection." Stevens (1950) stated, "We suppose that most people will find repugnant the idea of adding yet another random element to a result which is already subject to the errors of random sampling. But what one is really doing is to eliminate one uncertainty by introducing a new one. The uncertainty which is eliminated is that of the true probability that the parameter lies within the calculated interval. It is because this uncertainty is eliminated that we no longer have to keep 'on the safe side,' and can therefore reduce the width of the interval." He argued that "it is the statistician's duty to be wrong the stated proportion of times, and failure to reach this proportion is equivalent to using an inefficient in place of an efficient method of estimation." He noted, though, the apparent paradox

Alan Agresti is Distinguished Professor Emeritus, Department of Statistics, University of Florida, Gainesville, Florida 32611-8545, USA (e-mail: aa@stat.ufl.edu). Anna Gottard is Assistant Professor, Department of Statistics, University of Florence, Florence, Italy 50134 (e-mail: gottard@ds.unifi.it).