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Rejoinder: Fuzzy and Randomized Confidence Intervals and P-Values

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We thank all the discussants for their insightful comments. We enjoyed reading the historical background they supplied, were pleased by their new ideas for fuzzy procedures and were provoked to produce better arguments for our ideas (which is what comments are supposed to do).

1. NEW FUZZY PROCEDURES

We think the most illuminating aspect of the comments is the new fuzzy (or abstract randomized) procedures they propose.

1.1 Two New Binomial Fuzzy Confidence Intervals

Agresti and Gottard propose an equal-tailed fuzzy interval they attribute to Stevens (1950), although, of course, the notion of a *fuzzy* confidence interval was not exactly what Stevens proposed. This is the fuzzy confidence interval with membership function given by (1.1b) of our article, where ϕ is the critical function of the equal-tailed randomized test.

Brown, Cai and DasGupta propose a fuzzy interval they attribute to Pratt (1961), although, of course, the notion of a *fuzzy* confidence interval was not exactly what Pratt proposed. This is the fuzzy confidence interval with membership function given by (1.1b), where $\phi(\cdot, \alpha, \theta)$ is the critical function of the most powerful randomized simple-versus-simple test with null hypothesis that the data are Binomial(n, θ) and alternative hypothesis that the data have the discrete uniform distribution on $\{0, \dots, n\}$.

Figure 1 herein shows these two new fuzzy intervals along with the UMPU fuzzy intervals we proposed. Clearer and larger figures for more values of x are given on the web (`www.stat.umn.edu/geyer/fuzz`). From the figure it can be seen that the Pratt (Brown–Cai–DasGupta) intervals are not unimodal, a point noted by Pratt (1961) and by Brown, Cai, and DasGupta in their comments. These fuzzy intervals arise from an optimality argument we think shows a fundamental misunderstanding of fuzzy confidence intervals (which, of course, we cannot anachronistically blame Pratt for). From our point of view, what they actually do is optimally test against an alternative (discrete uniform) that we cannot imagine will ever be of interest in applications. Nevertheless, we say the more the merrier. If one likes these fuzzy intervals, then use them.

The equal-tailed (Agresti–Gottard) tests are more reasonable. There is little practical difference between their proposal and ours. As they say, their intervals look more reasonable for x in the middle of the range and ours look more reasonable elsewhere, but good frequentists cannot think this way (however natural it may be), since any frequentist property depends on averaging over all x.

1.2 UMPU Fuzzy Intervals Defended

Define the *coverage* at a point θ' of a fuzzy confidence interval (1.1b) when θ is the true parameter value to be

$$c(\theta, \theta') = E_\theta\{1 - \phi(X, \alpha, \theta')\}. \tag{1}$$

When $\theta = \theta'$, this is the left-hand side of (1.3) in our article.

The UMPU properties transferred to the language of confidence intervals are as follows:

(i) The interval is exact, that is,

$$c(\theta, \theta) = 1 - \alpha \quad \text{for all } \theta.$$

(ii) The interval has higher coverage for the true unknown θ than any other θ, that is,

$$c(\theta, \theta) \ge c(\theta, \theta') \quad \text{for all } \theta \text{ and } \theta'.$$

(iii) Subject to the constraints (i) and (ii), the interval has the lowest possible coverage for all nontrue θ, that is,

$$c(\theta, \theta') \le \tilde{c}(\theta, \theta') \quad \text{whenever } \theta' \ne \theta,$$

where $\tilde{c}$ is the coverage for any other fuzzy confidence interval satisfying (i) and (ii) with c replaced by $\tilde{c}$.

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